Progress report 2005

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Main Scope

We develop new field theoretical methods applicable in situations where conventional perturbative methods (Feynman diagrams) fail.

We mostly work in the framework of the lattice formulation of scalar and gauge theories, where one has to perform high-dimensional integrations over fields located at the sites or the links of the lattice.

We use the renormalization group method (which relates the behavior on small lattices to the behavior on larger lattices) and improve existing expansions (weak coupling, strong coupling). We try to control the large field configurations in the multidimensional integrals discussed above.

Motivations

Today, a quantitative treatment of the large distance effects in Quantum Chromodynamics (crucial for masses and decay constants) can only be done with Monte Carlo simulations. We need to go beyond that.

The short distance effects can be described by perturbation theory, however in recent perturbative QCD calculations, it is not easy to decide if next to next leading order corrections improve the accuracy of the final result.

The self-interactions of hypothetical scalar particles (Higgs etc..) are likely to require a treatment beyond perturbation theory.

We expect to bring higher standards of accuracy in quantum field theory and to be able to make predictions that can be compared with experiments which emphasize precision ($g-2$, hadronic width of the Z, etc...).

Recent work:

- We Developed modified perturbative methods where we introduce a large field cutoff and fix this new parameter using the strong coupling expansion. All the details were worked out in quantum mechanics for the anharmonic oscillator and for the one plaquette lattice gauge theory.
- We are extending the method to 4-dimensional QCD in the quenched approximation. We compared series expansions with Monte Carlo results. We studied the effect of a field cut on the average plaquette.
- We are constructing the nonlinear scaling variables (similar to action-angle) for renormalization group transformations in scalar field theory. This allows us to interpolate between regimes where different expansions are valid.

Current Plans

- Calculation of the non-perturbative part of the plaquette average in $4D$ QCD with the new perturbative method.
- Calculation of the $3D$ critical exponents with a modified Parisi method.
- Analytic methods for the $1/N$ expansion of the hierarchical model.
- Numerical tests of perturbative triviality and stability bounds for the Higgs, in the hierarchical approximation.
- Improvement of the hierarchical approximation.

Graduate Students

My graduate student Li Li earned his Ph. D. last Spring (May 2005).

I recruited a new graduate student, Daping Du, who arrived this Fall. He started doing calculations related to an optimized perturbative method in scalar field theory and is helping me to install the parallel computing software on our new cluster.

In the current year budget (March 2005-February 2006), I have 6.7 months of support for him and he has to earn the rest by being a Teaching Assistant. This could be a serious problem next year due to language requirements imposed by the College for second year graduate students. I request 9 months of support for him for next year (March 2006-February 2007).

Computational Facilities

In 2003, we built a 16 node cluster that has allowed us to create large numbers of gauge configurations on lattices with up to 16^4 sites and to work on scalar field theory in various dimensions. It was operated by Li Li who graduated in May 2005.

We have just finished building a new cluster with 8 single CPU nodes with 3.2 GHz Pentium 4 processors and Gigabyte motherboards with a build-in fast ethernet card. The parallel computing public software Oscar 4.2 is being installed by Daping Du.

Multiprocessors computers have been build commercially and are low maintenance items. We request funds to acquire one workstation with 4 CPU units to use for distributed calculations. This type of workstation costs approximately 5,000 dollars.

References

[1] L. Li and Y. Meurice, *A tractable example of perturbation theory with a field cutoff: The anharmonic oscillator*, *J. Phys. A* **38** (2005) 8139–8153.

[2] L. Li and Y. Meurice, *Asymptotically universal crossover in perturbation theory with a field cutoff*, hep-th/0507196, *J. Phys. A* (submitted).

[3] L. Li and Y. Meurice, *Lattice gluodynamics at negative g**2*, *Phys. Rev. D* **71** (2005) 016008.

[4] L. Li and Y. Meurice, *An example of optimal field cut in lattice gauge perturbation theory*, *Phys. Rev.* **D71** (2005) 054509.

[5] L. Li and Y. Meurice, *Effects of large field cutoffs in scalar and gauge models*, *Nucl. Phys. Proc. Suppl.* **140** (2005) 788–790.

[6] L. Li and Y. Meurice, *About a possible 3rd order phase transition at T = 0 in 4D gluodynamics*, hep-lat/0507034, *Phys. Rev. D* (submitted).

[7] L. Li and Y. Meurice, *Is there a 3rd order phase transition in 4D gluodynamics?*, Contributed to 23rd International Symposium on Lattice Field Field: Lattice 2005, Trinity College, Dublin, Ireland, 25-30 Jul 2005, hep-lat/0509096.

[8] J. J. Godina, L. Li Y. Meurice and B. Oktay, *High-accuracy critical exponents of $O(N)$ hierarchical sigma models*, hep-th/0511194, *Phys. Rev. D* (submitted).

Review Article on Global Aspects of the Renormalization Group

A proposal for a review article “Global Aspects of the Renormalization Group Flows of Dyson’s Hierarchical Model” has been accepted by Jour. of Phys. A. In Spring 2005, I worked on a detailed sketch of the article. It was forwarded to the Editorial board and received very positive comments. The expected date of completion is May 2006 .

Conferences and Workshop

I attended a conference focused on the interface between particle physics and cosmology (Miami 2004) where I gave a presentation the "Effects of Large Field Cutoff in Field Theory".

A workshop on the recent challenges of lattice field theory was organized by the Kavli Institute for Theoretical Physics located on the UCSB campus. Thanks to a faculty scholar award, I was able to participate to the workshop for three weeks in March 2005. I gave a talk "Role of Large Field Configurations in Perturbation Theory". It can be seen and heard online at http://online.itp.ucsb.edu/online/lattice05/meurice/.

I gave talks at the conference Lattice 2005 held at Trinity College in Dublin and at Miami 2005.

Work on Scalar Models

The limitations of perturbation theory are well understood for scalar field theories. Large field configurations have little effect on commonly used observables but are important for the average of large powers of the field and dominate the large order behavior of perturbative series. A simple way to remove the large field configurations consists in restricting the range of integration for the scalar fields in the path integral. The method produces convergent series in nontrivial cases.

The simplest quantum mechanical example where this method can be applied is the anharmonic oscillator. Recently, we have treated this example in complete detail. Quantum mechanics can be seen as quantum field theory with one time dimension and zero space dimension. However, we will use the usual x notation (instead of ϕ) in the following.

The anharmonic oscillator with a "field" cut x_{max}

L. Li an Y. Meurice J. Phys. A **38** 8139-8153 (2005)

$$H = \frac{p^2}{2} + V(x) \ ,$$

with

$$V(x) = \begin{cases} \frac{1}{2}\omega^2 x^2 + \lambda x^4 & \text{if} \quad |x| < x_{\max} \\ \infty & \text{if} \quad |x| \geq x_{\max} \end{cases}$$

$$E_0(x_{max}) = \omega \sum_{k=0}^{\infty} E_0^{(k)}(x_{max})(\lambda/\omega^3)^k \ ,$$

$$R_k(x_{max}) \equiv E_0^{(k)}(x_{max})/E_0^{(k)}(\infty) \ ,$$

finite radius of convergence: $\lambda_c \simeq 65 x_{max}^{-6}$

Analytical formulas for the modified coefficients are available for small and large x_{max}. At low order, they are valid over a wide range.

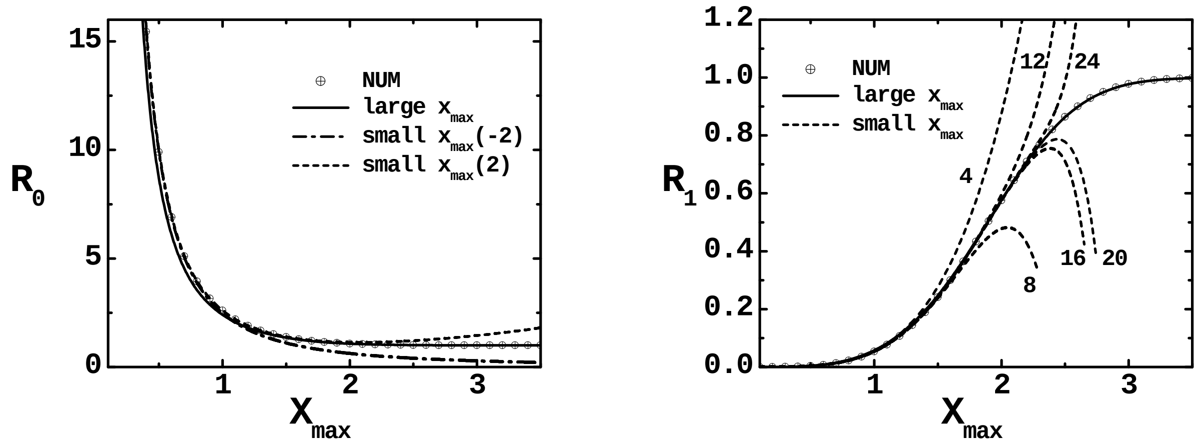


Figure 1: Numerical values of $R_0(x_{max})$ and $R_1(x_{max})$. The solid lines represent the large x_{max} expressions. The broken lines represent lowest orders in the small x_{max} approximation.

Universal shape of the modified coefficients as a function of the field cutoff

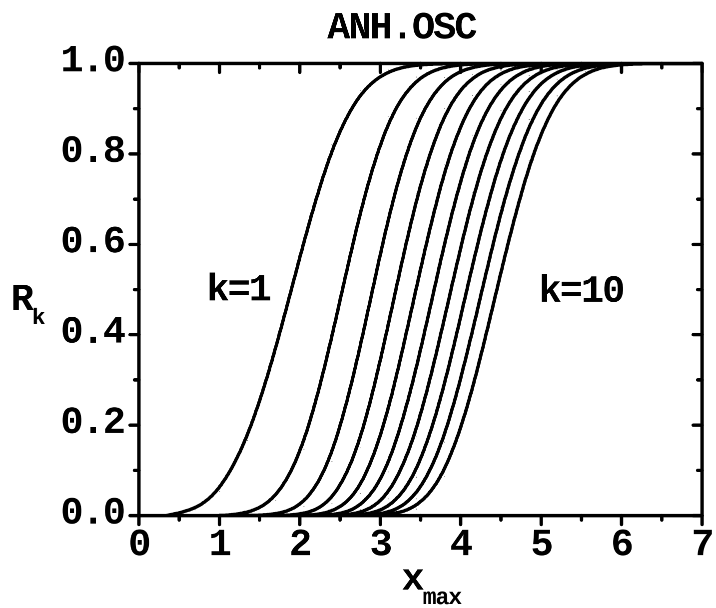


Figure 2: $R_k(x_{\rm max}) = E_0^{(k)}(x_{max})/E_0^{(k)}(\infty)$ for k going from 1 to 10.

Asymptotic data collapse

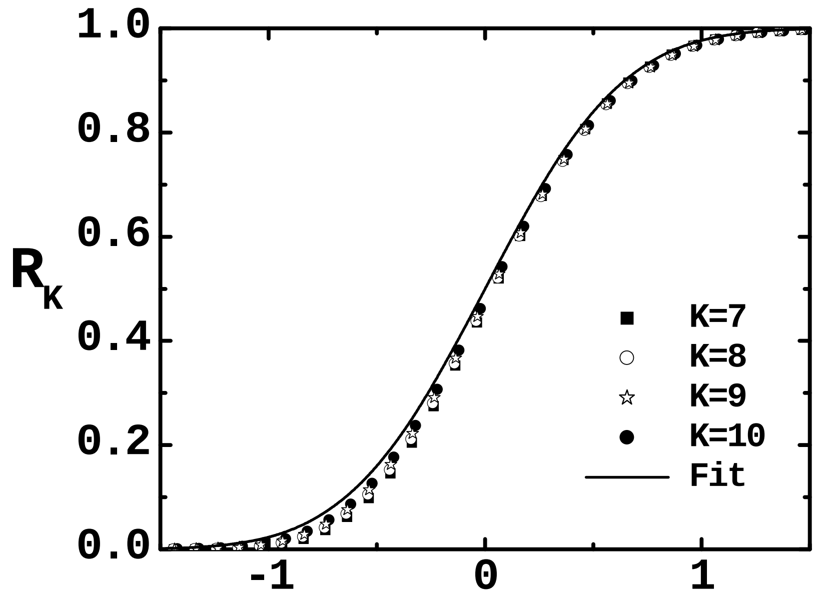


Figure 3: $R_k(x_+x_0(k))$ for $k = 7, \ldots 10$ and the function $U_{anh,1}(x)$.

Asymptotically universal behavior for the perturbative coefficients in the small-large field cutoff crossover

Li and Y. Meurice, J. Phys. A (submitted) hep-th/0507196

$$\ln(R'_k(x)) \simeq \sum_{q=0}^{Q} A_k^{(q)}(x - x_0(k))^q$$

$$A_k^{(2)} \simeq -1.5(1) + 2.0(1)k^{-0.6(1)} \ ,$$

$$A_k^{(3)} \simeq 1.3(3)k^{-0.9(1)} \ .$$

$$A_k^{(4)} \simeq -1.5(6)k^{-1.3(3)}$$

$$x_0(k) \simeq \sqrt{1.6k + 3.5}$$

Renormalization group picture

The beginning of the series corresponds to the behavior of the scaling variables near the gaussian fixed point.

The large order, corresponds to the approach of the high-temperature/strong-coupling fixed point.

The coefficients in the crossover (ϕ_{max} dependent) correspond to the crossover in the flows.

Tested using the hierarchical model.

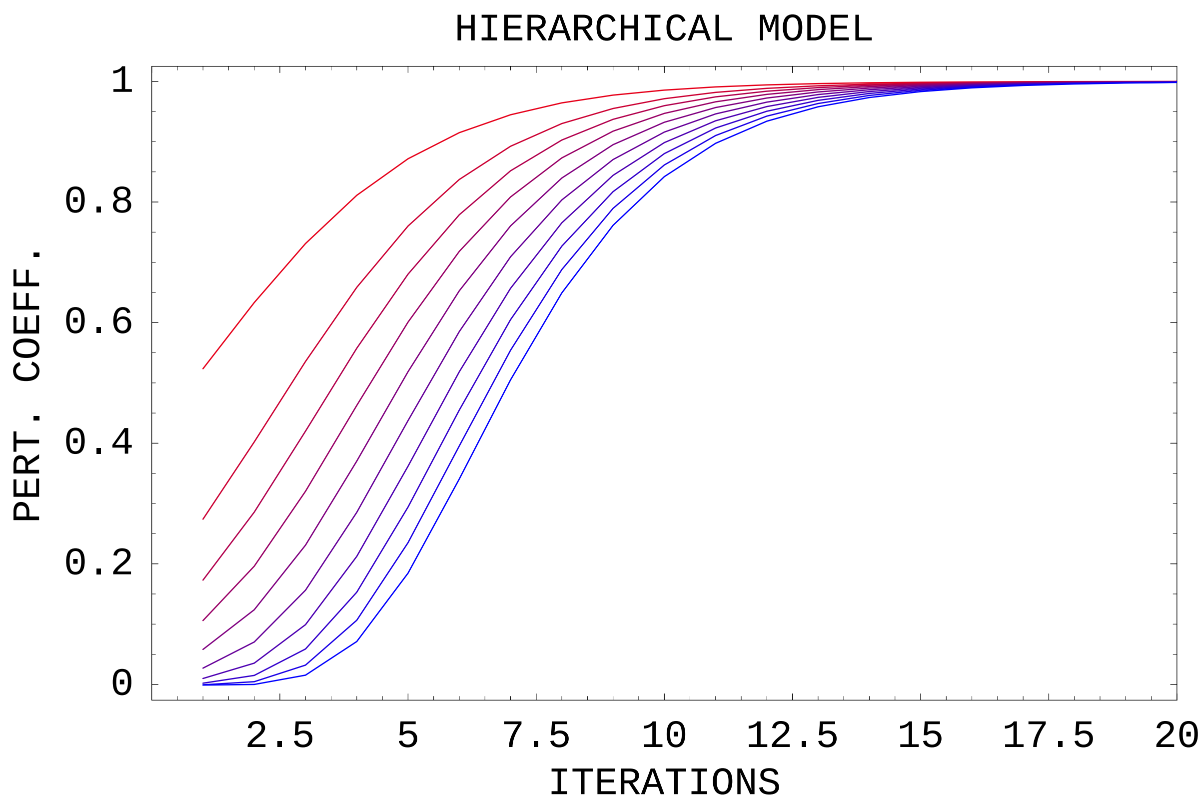


Figure 4: The first ten perturbative coefficients of the 2-point function of the hierarchical model with a field cut as a function of the number of iterations of the RG transformation. For comparison, they are given in unit of their asymptotic value. The curves represent the coefficients of order 1 to 10 from left to right (red to blue).

Optimization

The modified theory with a field cutoff differs from the original theory. Fortunately, it is possible to adjust the field cutoff to an optimal value in order to minimize or eliminate the discrepancy with the (usually unknown) correct value of the observable in the original theory. The strong coupling can be used to calculate approximately this optimal field cutoff. Examples can be found in articles published more than a year ago. In some cases, the expansion has a finite radius of convergence. Techniques to deal with this problem have been designed by an undergraduate student working in a R.E.U. program at the University of Iowa this summer 2005 Our next goal will be to calculate the three-dimensional critical exponents.

Borel summability of the $1/N$-expansion

The large N limit and the $1/N$ expansion appear prominently in recent developments in particle physics, condensed matter and string theory. In order to turn a $1/N$ expansion into a $quantitative$ tool, we need to: 1) understand the large order behavior of the series, 2) determine potential ambiguities of its Borel resummation and 3) compare the accuracy of various orders with numerical results for various N. We are answering these three questions for Dyson's hierarchical model. We provided high-accuracy numerical values for the critical exponent γ, the subleading exponent Δ and the critical parameter β_c for the $3D$ $O(N)$ hierarchical nonlinear sigma models for N up to 20. We are working on two independent methods to calculate the coefficient of the $1/N$ expansion of these critical quantities.

N	γ	Δ	β_c/N
1	1.29914073	0.425946859	1.179030170
2	1.41644996	0.475380831	1.236763288
3	1.52227970	0.532691965	1.275794011
4	1.60872817	0.590232008	1.302790391
5	1.67551051	0.642369187	1.322083069
6	1.72617703	0.686892637	1.336351901
7	1.76479863	0.723880426	1.347244235
8	1.79469274	0.754352622	1.355791342
9	1.81827105	0.779508505	1.362657559
10	1.83722291	0.800424484	1.368284407
11	1.85272636	0.817977695	1.372974325
12	1.86561092	0.832855522	1.376940318
13	1.87646998	0.845589221	1.380336209
14	1.88573562	0.856588705	1.383275590
15	1.89372812	0.866171682	1.385844022
16	1.90068903	0.874586271	1.388107107
17	1.90680338	0.882027998	1.390115936
18	1.91221507	0.888652409	1.391910870
19	1.91703752	0.894584429	1.393524199
20	1.92136121	0.899925325	1.394982051
∞	2	1	$\frac{2-c}{2(c-1)} = 1.42366..$

Work on Lattice Gauge Models

We are extending the method for lattice gauge theories. For compact groups such as $SU(N)$, the gauge fields are not arbitrarily large. It is possible to define a sensible theory at negative $\beta = 2N/g^2$. We plan to work on the phase diagram for negative coupling for the adjoint part of the action.

We have compared the effects of local and nonlocal field cuts. For scalar fields, the configurations can be ranked according to the largest absolute value of the field or according to the average (so this is non-local in configuration space) over the sites of an even power of the field. The largest this power is, the more emphasis is put on the configurations with the largest field values. We observed correlations among these quantities in the scalar case.

We followed the same procedure for lattice gauge models using the Landau gauge where $1-(1/N)ReTrU_{link}$ plays a role analogous to ϕ^2 in scalar models. We found correlations between the lattice average of this quantity and the average action. We found correlations between the average and the maximum value when the Landau gauge condition is implemented carefully .

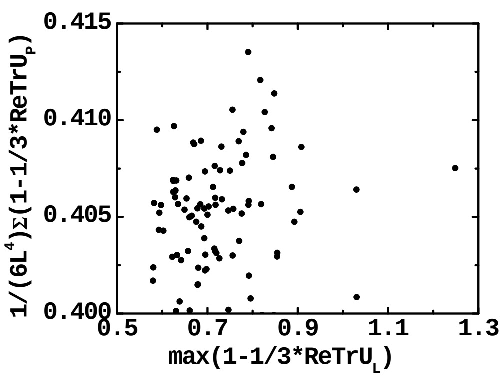


Figure 5: The picture is the average plaquette as a function of $Max\{1-\frac{1}{3}ReTrU_L\}$.

One plaquette LGT

L. Li, Y. Meurice, Phys. Rev. D **71** 054509 (2005)

$$Z(\beta, N) = \int \prod_{l \in p} dU_l \mathrm{e}^{-\beta(1-\frac{1}{N} ReTrU_p)} \ ,$$

$$Z(\beta, 2) = (2/\beta)^{3/2} \frac{1}{\pi} \int_0^{2\beta} dt t^{1/2} \mathrm{e}^{-t} \sqrt{1-(t/2\beta)}$$

modified partition function:

$$Z(\beta, 2, t_{max}) = (2/\beta)^{3/2} \frac{1}{\pi} \int_0^{t_{max}} dt t^{1/2} \mathrm{e}^{-t} \sqrt{1-(t/2\beta)}$$

$$Z(\beta, 2, t_{max}) = (\beta\pi)^{-3/2} 2^{1/2} \sum_{l=0}^{\infty} A_l(t_{max})(2\beta)^{-l} \ ,$$

with

$$A_l(t_{max}) \equiv \frac{\Gamma(l+1/2)}{l!(1/2-l)} \int_0^{t_{max}} dt \mathrm{e}^{-t} t^{l+1/2} \ ,$$

When $t_{max} \rightarrow \infty$ the integral becomes the (complete) Γ function and the coefficients grow factorially. In lattice perturbation theory, we "add the tails".

Note: $t_{max} = 2\beta$ means β-dependent coefficients.

When t_{max} is finite, the integral is bounded by a power of t_{max}. When $t_{max} \leq 2\beta$, the sum converges.

The need for interpolation

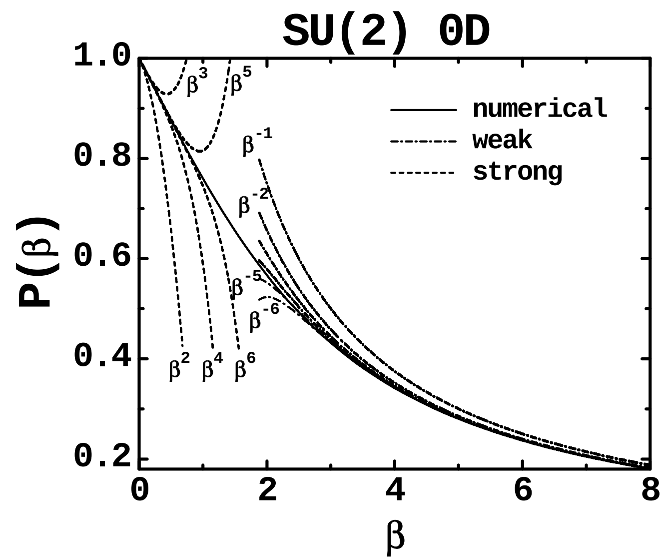


Figure 6: P versus β for $SU(2)$ on one plaquette. The solid line represents the numerical values; the dashed lines on the left, successive orders in the strong coupling expansion; the dot-dash lines on the right, successive order in the weak coupling expansion.

Optimal t_{max}

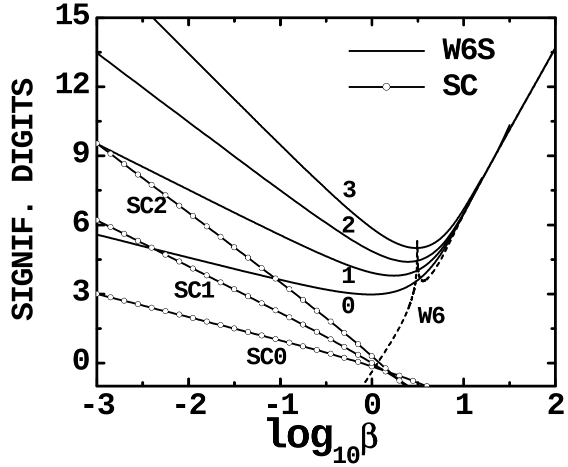


Figure 7: Significant digits obtained from the weak series truncated at order 6, calculating t_{max}/β using the strong coupling expansion at order 0 to 3, compared to the weak coupling expansion at order 6 (dotted line W6) and the strong coupling expansion at order 0 to 2 (empty circles SC). Hopefully it can be extended to $4D$ where calculations are much harder!

$4D$ Quenched QCD

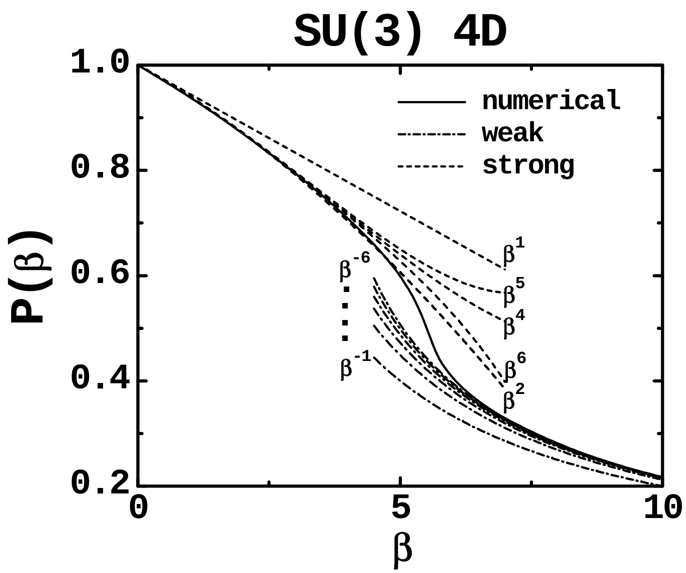


Figure 8: P versus β for $SU(3)$ in 4 dimensions. The solid line represents the numerical values; the dashed lines on the left, successive orders in the strong coupling expansion; the dot-dash lines on the right, successive orders in the weak coupling expansion. $P \sim < F^a_{\mu\nu} F^{a\mu\nu} >$, the non-perturbative part is often called gluon condensate.

Discontinuity at $g^2 \to \pm 0$

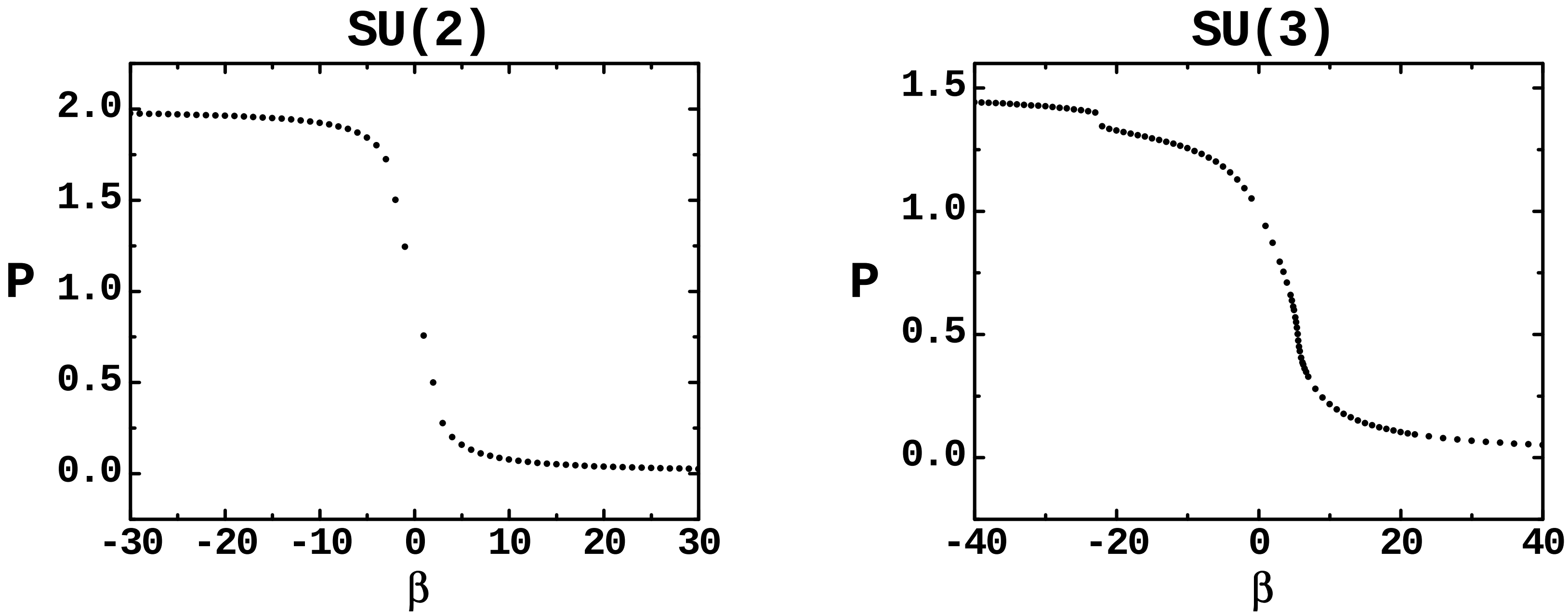
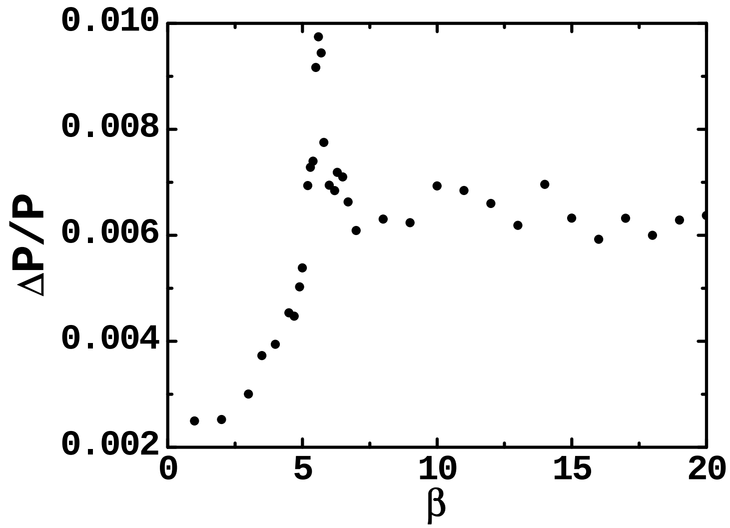


Figure 9: MC calculation of the average action density $P(\beta)$ for $SU(2)$ and $SU(3)$ L. Li and Y. Meurice, Phys. Rev. D **71** 016008 (2005).

Effect of a gauge invariant cut on P

The effect of the cut is very small but of a different size below, near or above $\beta = 5.6$. The relative change of the configuration average of P when 80 percent of the large field configurations are discarded, for various values of β in a pure $SU(3)$ LGT on a 8^4 lattice is shown below (see L. Li and Y. Meurice, Nucl. Phys. Proc. Suppl. **14** 788-790 (2005)).



Lattice Perturbation Theory

We used the series of Di Renzo et al. for the average plaquette.

$$P(1/\beta) = \sum_{m=0}^{10} b_m \beta^{-m} + \ldots .$$

$r_m = b_m/b_{m-1}$, the ratio of two successive coefficients extrapolates near 6 when $m \to \infty$). On the other hand, we expect a linear growth for an asymptotic series.

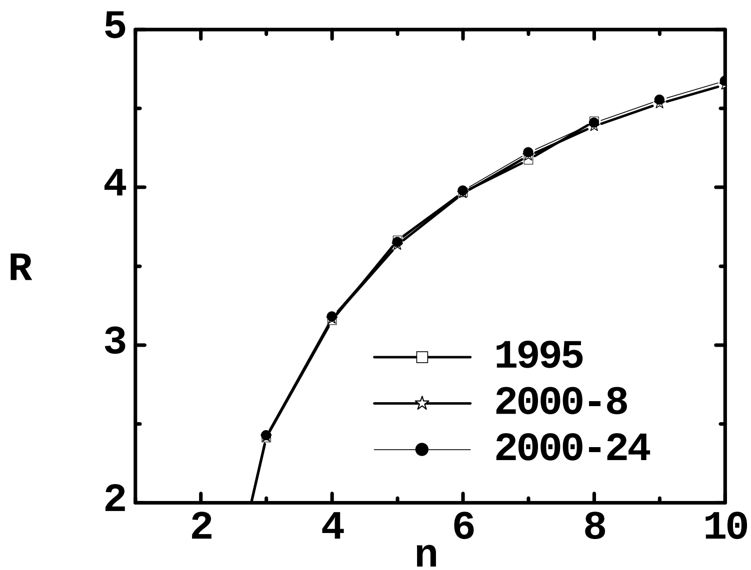


Figure 10: Ratios for the 95 and 2000 data (on 8^4 and 24^4 lattices).

This suggests $P = (1/\beta_c - 1/\beta)^{1-\alpha}$. This implies a massless glueball! (not seen). No hint of modulations due to complex singularities.

Series Analysis

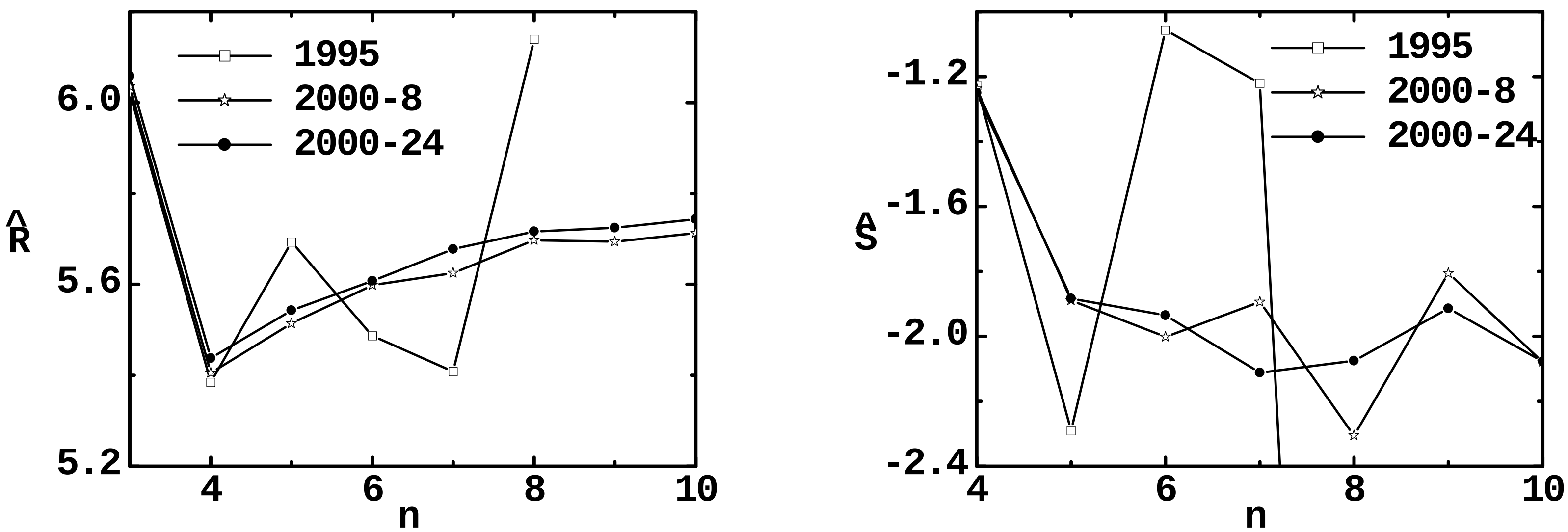


Figure 11: The extrapolated ratios (left) suggests $\beta_c \simeq 5.74$. The extrapolated slope (right)suggests $-2 + \alpha = -2.08$.

$$P_{non-pert.} = (P - P_{pert.}) \propto a^4 \propto \left(\mathrm{e}^{-\frac{16\pi^2}{33}\beta} \right)$$

Fitting in the interval $5.6 < \beta < 6.0$: $P_{non-pert.} \simeq 1.2 \times 10^{10} \times \mathrm{e}^{-\frac{16\pi^2}{33}\beta}$

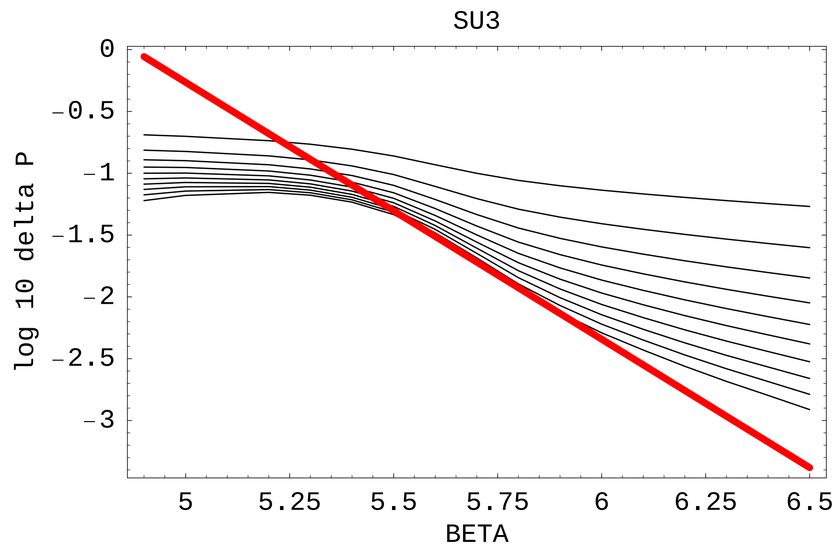


Figure 12: $Log_{10}|P - P_{pert.}|$ for order 1 to 10 ; the a^4 fit (red).

Direct Search for Singularities in P' and P''

$$-\partial P/\partial\beta = (1/\mathcal{N}_p)[\left\langle\Sigma^2\right\rangle - \langle\Sigma\rangle^2] \, , \tag{1}$$

$$\partial^2 P/\partial\beta^2 = (1/\mathcal{N}_p)[\left\langle\Sigma^3\right\rangle - 3\langle\Sigma\rangle\left\langle\Sigma^2\right\rangle + \langle\Sigma\rangle^3] \tag{2}$$

Loss of precision in the calculation of the higher moments: in $-\partial P\partial\beta$, the two terms are of order $\mathcal{N}_p$ but their difference is of order 1. For 10^4 lattice, P'' will appear in the ninth significant digit and the use of double precision is crucial.

the peak disappears on L^4 lattices

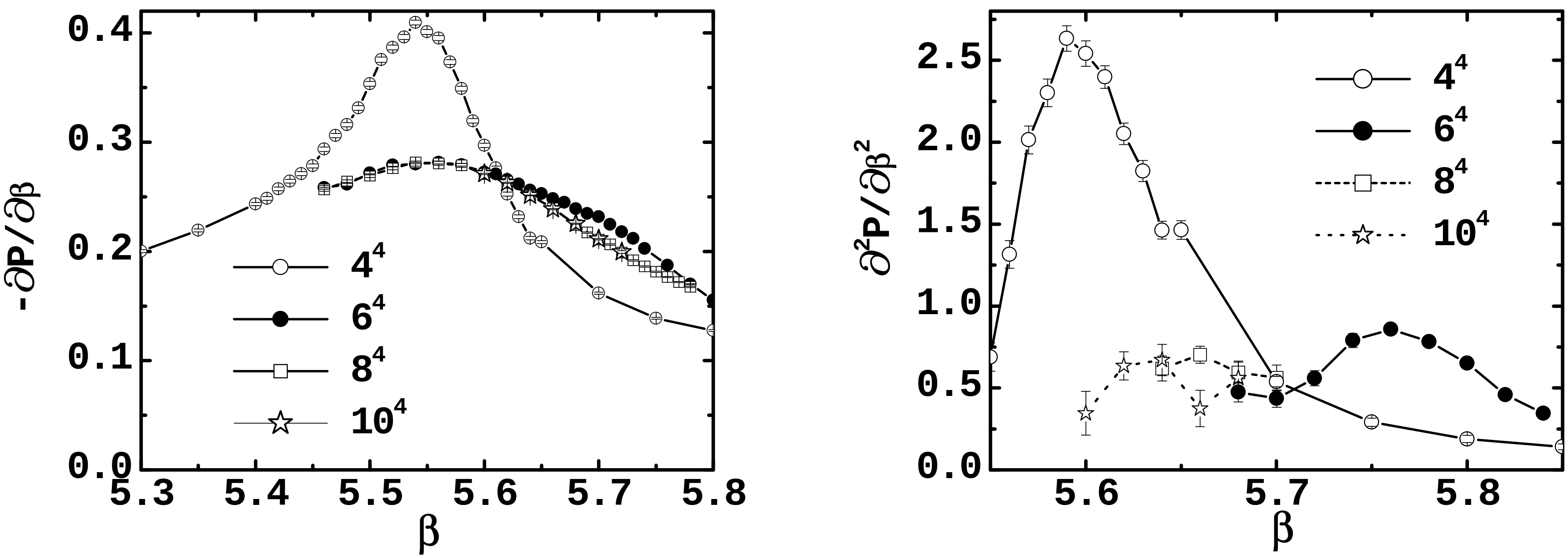


Figure 13: First and second derivative of P versus β on L^4 lattices. Two months on a 16 node cluster!

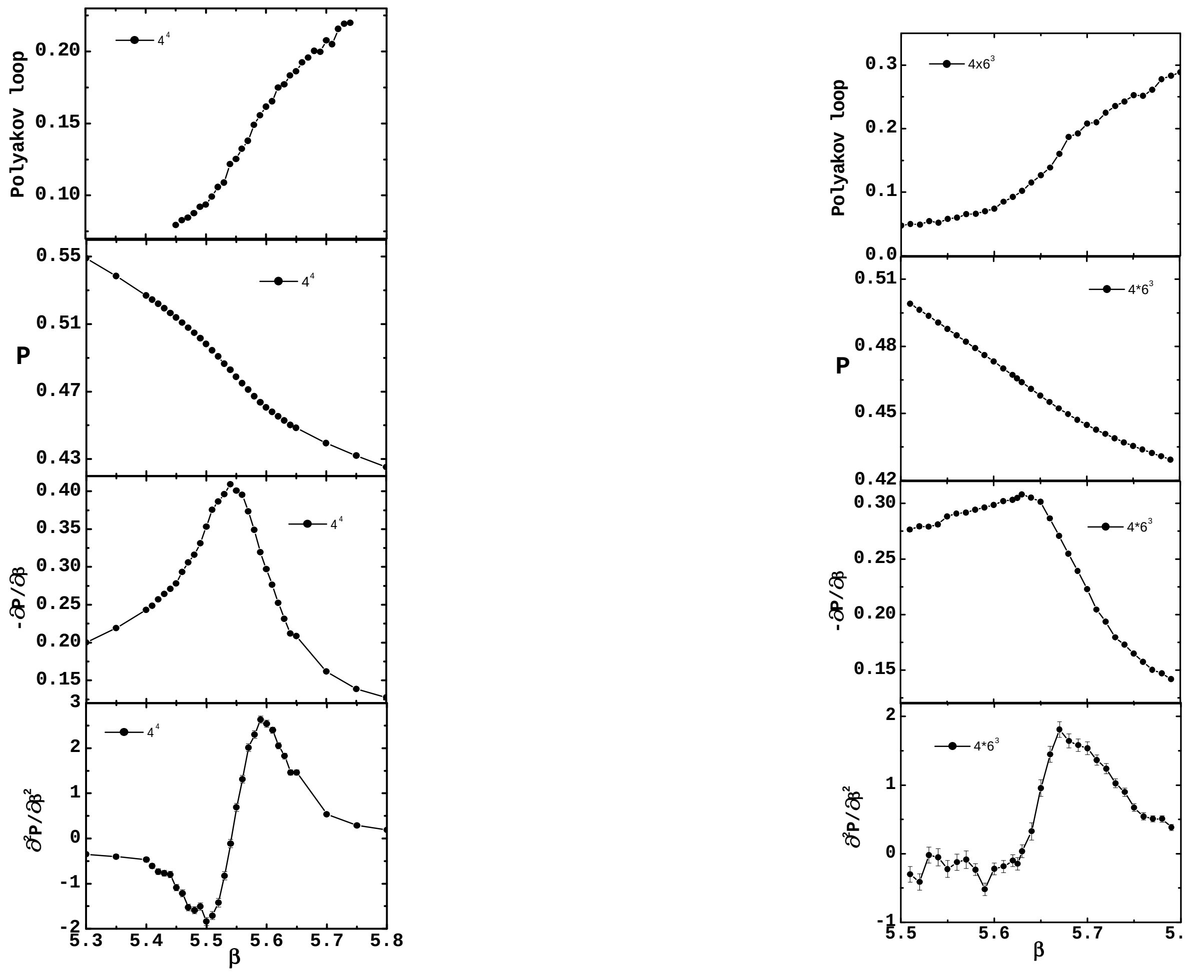


Figure 14: The Polyakov loop, P, first and second derivative of P versus β for 4^4 and 4×6^3 lattices .The peak is related to the finite temperature transition.

A simple alternative can be designed by assuming that the critical point in the fundamental-adjoint plane has mean field exponents and in particular $\alpha = 0$. We will further assume an approximate logarithmic behavior

$$-\partial P/\partial\beta \propto \ln((1/\beta_m - 1/\beta)^2 + \Gamma^2) \ , \tag{3}$$

on the axis where the adjoint term of the action is zero (the range of parameters considered here). $1/\beta_m$ denotes the value where the argument of the logarithm is maximal on this axis. This implies the approximate form

$$\partial^2 P/\partial\beta^2 \simeq -C\frac{(1/\beta_m - 1/\beta)}{\beta^3((1/\beta_m - 1/\beta)^2 + \Gamma^2)} \tag{4}$$

The β^3 at the denominator ensures that the series starts at β^{-3}.

Fits: $\beta_m \simeq 5.78$, $\Gamma \simeq 0.006$, and $C \simeq 0.15$

The variations in the estimation of C (typically $|\delta C| \sim 0.01$) and β_m (typically $|\delta\beta_m| \sim 0.02$) are small. On the other hand, Γ varies more rapidly under changes of the weights in the χ^2 function. We found values of Γ between 0.003 and 0.007.

The stability of C and β_m can be used to set a lower bound on Γ. Given that the approximate form of $\partial^2 P/\partial\beta^2$ in Eq. (4) has extrema at $1/\beta = 1/\beta_m \pm \Gamma$. As we do not observe values larger than 0.3 near $\beta = 5.75$ (see Fig. 13) we get the approximate bound

$$\frac{C}{2\beta_m^3\Gamma} < 0.3 \tag{5}$$

This implies the lower bound $\Gamma > 0.001$. On the other hand, large values of Γ are also excluded. We never found estimate close to 0.01.

Small window for a complex singularity

The imaginary part Γ of the location of the singularity in the $1/\beta = g^2/6$ plane is

$$0.001 < \Gamma < 0.01 \ . \tag{6}$$

We also performed calculations with an assumption similar to Eq. (4) but with $((1/\beta_m - 1/\beta)^2 + \Gamma^2)^{1+(\alpha/2)}$ at the denominator, for small positive and negative values of α. We found very similar ranges of values for the unknown parameters and we were able to draw very similar conclusions as for $\alpha = 0$.

A puzzling aspect of Fig. 13 is that the maximum of the first and second derivatives are not located near 5.78 but near lower values (5.55 and 5.63 respectively). This can be explained quantitatively from the non-perturbative part (see hep-lat/0507034)

Remarks

Good support for “decent flows make decent series”

A lot of work remains to be done: construction of gaussian scaling variables, optimization methods in presence of complex singularities, modified coefficients for quenched QCD.

Proving confinement in the continuum limit ($\beta \to \infty$) can't be done with simulations (the physical size of the lattice becomes too small to have large Wilson's loops). We need analytical methods. If you can also prove the existence of a mass gap, you may become a millionaire! (one of the 10 millennium problems of the Clay institute)