Theoretical Physics at the University of Iowa
Part 1: New Methods in Field Theory

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December 16, 2003

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Overview

- Our main activity is the development of new field theoretical methods which can be used in situations where conventional perturbative methods fail or where lattice methods have a limited accuracy.

- Our long term goal is to bring higher standards of accuracy in quantum field theory and to be able to make predictions that can be compared with experiments which emphasize precision ($g-2$, hadronic width of the Z, etc...).

- A central theme in our research program is the interpolation between scales where different approximations are available (example: Feynman diagrams at short distance and strong coupling expansion at large distance).

Remarks

- A natural tool to interpolate between different scales is the renormalization group method.

- In order to fully establish a standard model of strong and electroweak interactions, nonlinear aspects of the renormalization group (related to confinement, chiral symmetry breaking and mass generation) need to be understood.

- Nonlinear aspects of the RG flows are difficult and in a first approach simplified models need to be considered (e. g., Dyson's Hierarchical Model).

Students Involved

- B. Oktay: supported until summer 2001 (Ph. D.); postdoc at the University of Illinois at Urbana Champaign.

- L. Li: passed the qualifying exam and the comprehensive exam; recipient of the Goertz-Nicholson award in May 2001; supported by the grant as a RA; Played a major role in designing and building our 16 node cluster.

- Brian Kessler and Andrew Lytle: undergraduate students; supported by a Undergraduate Scholar Assistantship from the University (A.L. has met the 2 year limit); Co-recipients of the Van Allen research award in summer 2003.

Computer Facilities: Linux Cluster

- 16 single CPU nodes; 1.6Ghz AMD Athlon XP processor. The maximal LINPACK performance achieved is 17.95 GFlop (number 8 in 1993!)

- Networking: 10M/100M fast Ethernet on each node. The maximum bandwidth is 85.08Mbps when the packet size is 49152 bytes, and the corresponding latency is 0.004408 second.

- Infrastructure paid by the University, computer parts by the D.O.E. .

- Administered by L. Li

MILC QCD Code Benchmarks

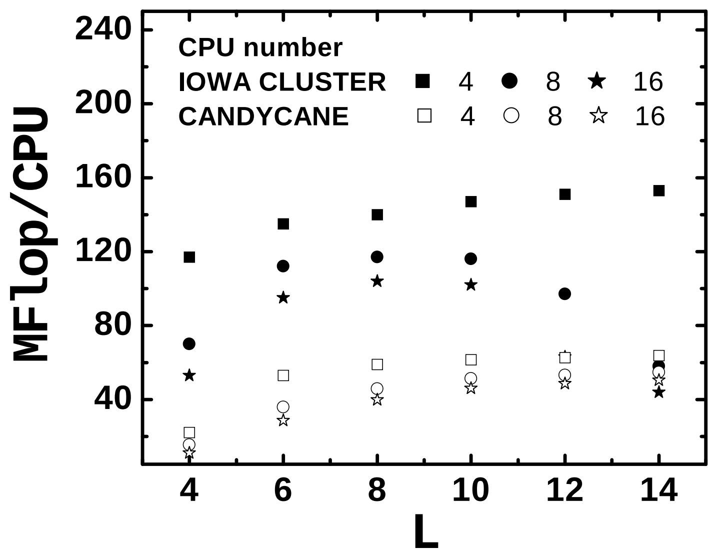

Figure 1: Comparison with CANDYCANE (16 350MHz nodes, Fast Ethernet). The problem size is L^4.

1. Interpolation between fixed points of the renormalization group transformation

- There exists a close connection between statistical mechanics near criticality and Euclidean field theory in the large cut-off limit (Wilson 71).

- The determination of the renormalized quantities at zero momentum amounts to the determination of a certain number of parameters appearing in the scaling laws. Some of these parameters are universal (the critical exponents) and much effort has been successfully devoted to their calculation. On the other hand, new techniques need to be developed in order to reliably calculate the non-universal parameters (critical amplitudes).

For values of β slightly smaller than β_c, we have

$$\chi^{(2l)} \simeq (\beta_c - \beta)^{-\gamma_{2l}} \left[A_0^{(2l)} + A_1^{(2l)} (\beta_c - \beta)^{\Delta} \right.$$
$$\left. + \quad A_{per.}^{(2l)} \cos\left(\omega \ln(\beta_c - \beta) + \phi^{(2l)}\right) + \ldots \right] , \qquad (1)$$

with known parameters γ_{2l}, Δ (not to be confused with the gap exponent) and ω. We have calculated these values for the hierarchical model:

$$\gamma_{2l} = \gamma(5l - 3) , \qquad (2)$$

with

$$\gamma \simeq 1.299140730159 \qquad (3)$$

and

$$\Delta \simeq 0.42595 . \qquad (4)$$

Example: the renormalized mass

$$m_R^2 = \frac{\Lambda_R^2 u^\gamma}{A_0^{(2)} + A_1^{(2)} u^\Delta (\frac{\Lambda_R}{\Lambda_L})^{\frac{2\Delta}{\gamma}} + LPC + \ldots} \qquad (5)$$

with the log-periodic corrections

$$LPC = A_{per.}^{(2)} \cos\left(\omega\left(\ln u + \frac{2}{\gamma}\ln\left(\frac{\Lambda_R}{\Lambda_L}\right)\right) + \phi^{(2)}\right) \ . \qquad (6)$$

– Note that the existence of a limit cycle (effect of the discreteness of the RG) but $A_{per.}^{(2)}$ is very small.
– Recent examples of limit cycles: Braaten et al. PRL 91 102002 (20030); Glazek and Wilson PRL 89 230401 (2002).
– The amplitudes need to be calculated!

- The calculation of the critical amplitudes requires a detailed representation of the RG flows. This is a difficult nonlinear problem. A common strategy in problems involving nonlinear flows near a singular point, is to construct a new system of coordinates for which the governing equations become linear. In the context of the RG, these variables are called the scaling variables or nonlinear scaling fields.

- We proposed to combine the nonlinear scaling fields associated with the high-temperature (HT) fixed point, with those associated with the unstable fixed point, in order to construct the critical amplitudes as RG invariants

- For more details see: Y. Meurice and S. Niermann, From Nonlinear Scaling Fields to Critical Amplitudes, J. Stat. Phys. 108, 213 (2002).

- The construction of the nonlinear scaling fields associated with the HT fixed point, suffer from an apparent small denominator problem, however a small numerator mechanism has been observed.

- For detail see: Y. Meurice, Phys. Rev. E **63**, 055101 (Rapid Communication) (2001).

- A general explanation where dimensional regularization is used to observe the cancellation is given. If log-periodic corrections are neglected, the dimensionless renormalized coupling constants

$$\zeta^{(2l)} \propto \chi^{(2l)}(\beta_c - \lambda_1^{-L} u)\, m_R^{2l(1+D/2)-D} \ . \tag{7}$$

  take universal values (conserved charges).

- For detail see: Y. Meurice, Phys. Rev. E (submitted).

- The values of the universal couplings have been calculated

$2l$	$\zeta^{(2l)\star}$
4	1.505871
6	18.1072
8	579.970
10	$35654.$
12	$3.5777\,10^{6}$
14	$5.318\,10^{8}$
16	$1.097\,10^{11}$
18	$3.00\,10^{13}$
20	$1.05\,10^{16}$

- For detail see: Y. Meurice and B. Oktay, U. of Iowa preprint (draft)

- In the inifinite cut-off limit, the only free parameter is the amplitude of the 2-point function (predictivity without fine-tuning).

- The values of the coupling appear consistent with the $Al!\exp(Bl)l^C$ behavior found in multiparticle production.

- The scaling variables provide very efficient infinite volume extrapolations (applicable generically).

- A more general discussion of the nonlinear properties of the RG flow will appear in a review article. A proposal for this review article has been accepted by Journal of Physics A.

2. The large-N approach of $O(N)$ sigma models

- The conventional expansion in powers of the field for the critical potential of 3-dimensional $O(N)$ models in the large-N limit, does not converge for values of ϕ^2 larger than some critical value.

- Padé approximants $[L+3/L]$ for the critical potential apparently converge at large ϕ^2. This allows high-precision calculation of the fixed point in a more suitable set of coordinates.

- We found numerical evidence for conjugated branch points in the complex ϕ^2 plane (see figure below). Ignoring these singularities may lead to inaccurate approximations.

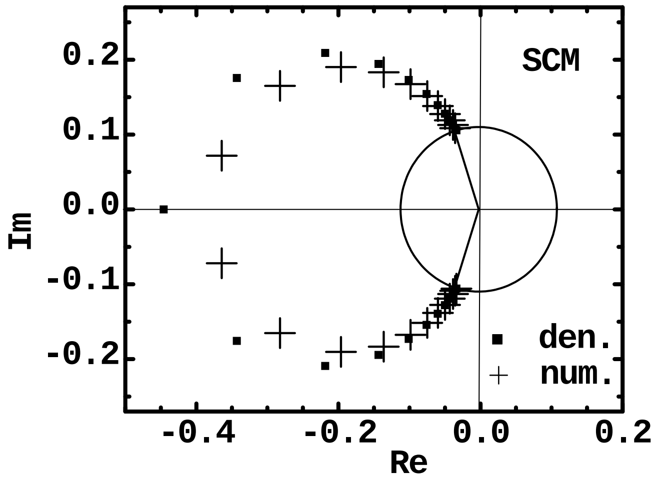

Figure 2: Real and imaginary parts of the roots of the denominator (filled squares) and numerator (crosses) of a [26/23] Padé approximant for the critical potential.

- More details in: Y. Meurice, Phys. Rev. D 67 025006 (2003).

Work in Progress, Plans:

- We have studied the critical properties (fixed points, exponents) of $O(N)$ hierarchical non linear sigma models at values of N ranging from 1 to 130. This work involved J. J. Godina, B. Oktay and L. Li. We are presently attempting to determine the first four coefficients of the $1/N$ expansion in order to get some idea about the Borel summability of the series. Preliminary results seem to favor the Borel summability.

- Example:

$$\beta_c = N\frac{2-c}{2(c-1)}\left(1-\frac{0.4150}{N}+\frac{0.2263}{N^2}+\frac{0.3233}{N^3}+\frac{0.3156}{N^4}\right) \qquad (8)$$

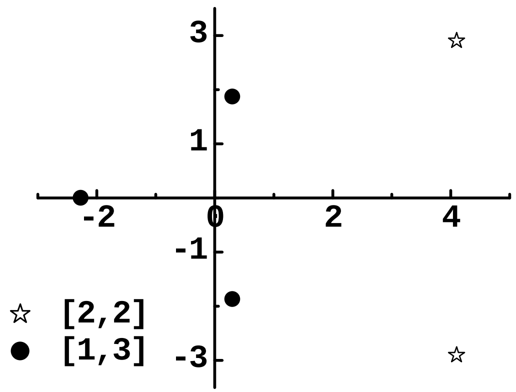

Figure 3: The poles of two Padé approximants for the Borel sum

- We have observed that in a system of coordinates where the unstable fixed point can be approximated by polynomials, the procedure which consists in considering bare potential truncated at order $(\phi^2)^3$ has a low accuracy.

- We are planning to investigate if similar problems appear near tricritical fixed points. In particular, reconsidering the RG flows in a larger space of bare parameters may affect the generic dimension of the intersections of hypersurface of various codimensions and help us finding a more general realization of spontaneous breaking of scale invariance with a dynamical generation of mass as in the Bardeen-Moshe-Bander mechanism.

3. Optimized Perturbative Expansions

- For two non-trivial $\lambda\phi^4$ problems (anharmonic oscillator and hierarchical model) improved perturbative series can be obtained by cutting off the large field contributions.

- The modified series converge to values exponentially close to the exact ones.

- For λ larger than some critical value, the method outperforms Padé's approximants and Borel summations.

- The method can be used for series which are not Borel summable such as the double-well potential series. (QCD is not Borel summable)

Figure 4: Number of significant digits for the double-well at order 3 to 6 for regular perturbation (black) compared to series obtained with $y_{min} = -3$ and $y_{max} = 3$ (blue) or $y_{max} = 2.5$ (green). As the order increases, the black curves reach the one-instanton contribution (red) over wider regions to the left while the two other sets reach the accuracy level obtained numerically for $y_{max} = 3$ (purple) or $y_{max} = 2.5$ (brown).

- More details in: Y. Meurice, Simple Method to Make Asymptotic Series of Feynman Diagrams Converge, Phys. Rev. Lett. 88, 141601 (2002).

- We are developing various methods to calculate the coefficients of the modified perturbative series.

- Our goal is to obtain more accurate estimates of the critical exponents in 3 dimensions (using Parisi's method).

- For one dimensional problems, we found a perturbative version of the accurate numerical methods which motivated this approach. (Y. Meurice, Arbitrarily Accurate Eigenvalues for One-dimensional Polynomial Potentials, J. Phys. A 35, 8831 (2002)).

- We were able to reproduce these accurate results using the Monte Carlo method. (L. Li and Y.Meurice, draft in progress)
  - We used a mixed (Metropolis+Overrelaxation) method that is very efficient at small lattice spacing (we want to minimize the CPU time between uncorrelated configurations).

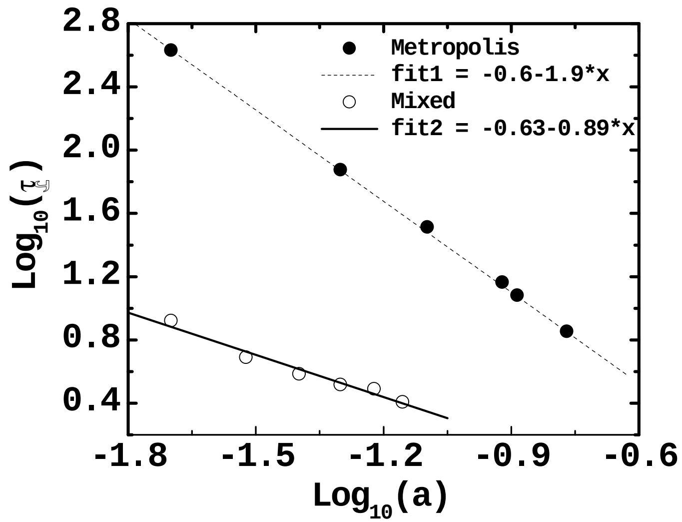

Figure 5: The logarithm of the correlation time versus the logarithm of lattice spacing for the Metropolis and mixed methods.

– Nonlinear fits to get the continuum limit. Example: the third coefficient; note the small statistical noise despite two subtractions! (Here the asymptotic nature of the series is indeed helpful)

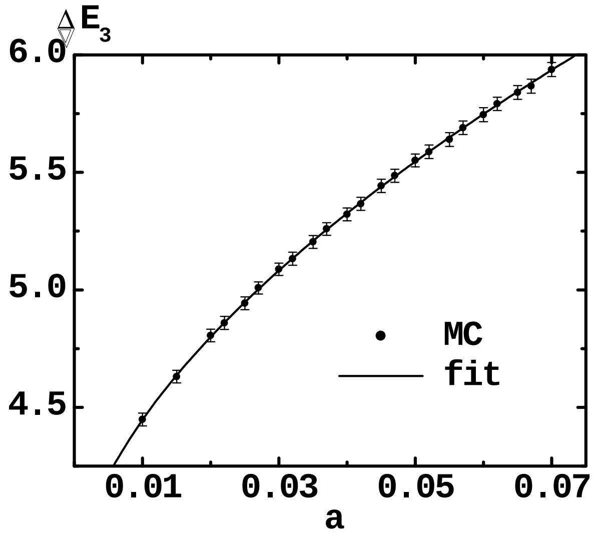

Figure 6: The Monte Carlo result (circles) and fitting result (continuous line) of ΔE_3 when $x_{\max} = 2.5$. The accurate result of ΔE_3 when $x_{\max} = 2.5$ is 3.649. And the fitting function is $3.730 + 19.25a^{0.578}$.

– Comparison with accurate results

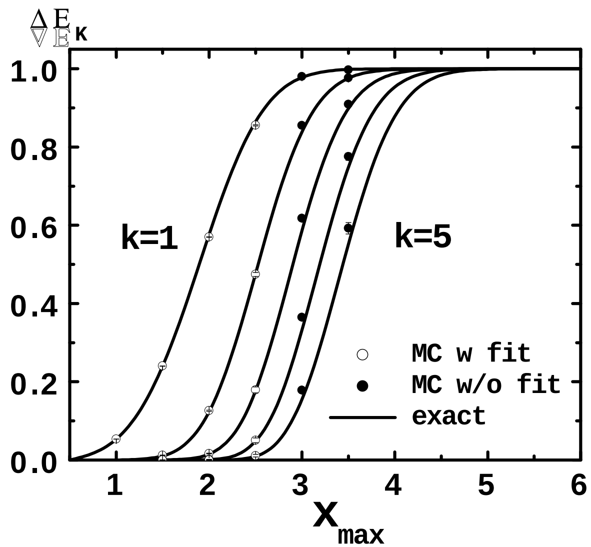

Figure 7: The comparison of Monte Carlo result (circles) and accurate numerical results (continuous line), all the values has been divided by their infinite cut limits

- For $D = 2$ and 3, the field cutoff is also a UV cutoff!

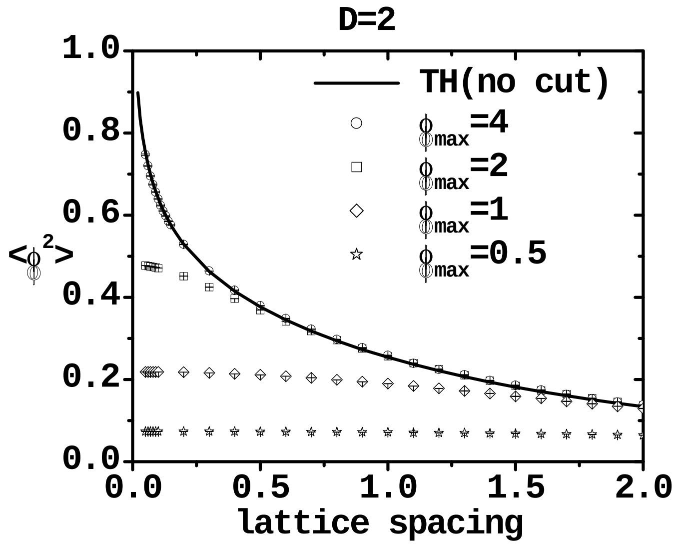

Figure 8: $< \phi^2 >$ for $D = 2$

Optimization:
What is the best field cut?

- For the simple integral

$$Z(\lambda) = \int_{-\infty}^{+\infty} d\phi \mathrm{e}^{-(m^2/2)\phi^2 - \lambda\phi^4} . \tag{9}$$

the perturbative series terminated at even order and calculated with a field cut ϕ_{max} provides the exact answer provided that we adjust ϕ_{max} properly. The underestimation due to the cut can be compensated by the overestimation due to the truncation at even order! Approximate values of ϕ_{max} can be calculated using the strong coupling expansion

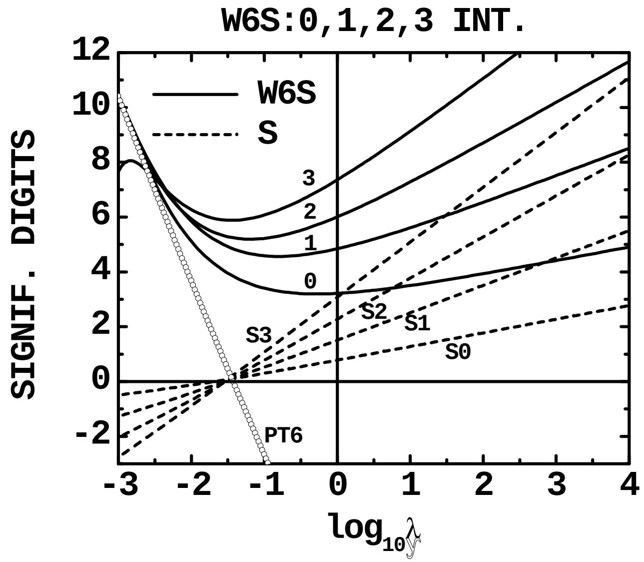

Figure 9: Significant digits obtained with the optimal cut $\phi_{max}(\lambda)$ (corresponding to a truncated expansion at order 6 in the weak coupling) estimated using the strong coupling expansion at orders 0, 1, 2 and 3 (solid lines), compared to significant digits using only the strong coupling expansion of the integral at the same orders in the strong coupling (dashed lines) and regular perturbation theory at order 6 (PT6).

- For detail see: B. Kessler, L. Li and Y. Meurice, Phys. Rev. D. (January 2004), hep-th/0309022.

- Generalization for the ground state of the anharmonic oscillator in progress. The under/over estimation pattern is more involved.

Interpolation between the large and small field

(L. Li and Y. Meurice, draft in progress)

Basic idea:

$$\begin{aligned}
\frac{a_k(\phi_{\rm max})}{a_k(\infty)} &= \frac{\int_0^{\phi_{\rm max}} d\phi e^{-(m^2/2)\phi^2}\phi^{4k}}{\int_0^{\infty} d\phi e^{-(m^2/2)\phi^2}\phi^{4k}} \\
&\simeq \frac{\int_{-\infty}^{\phi_{\rm max}} d\phi e^{-(m\phi-\sqrt{4k})^2}}{\sqrt{\pi/m^2}}
\end{aligned} \tag{10}$$

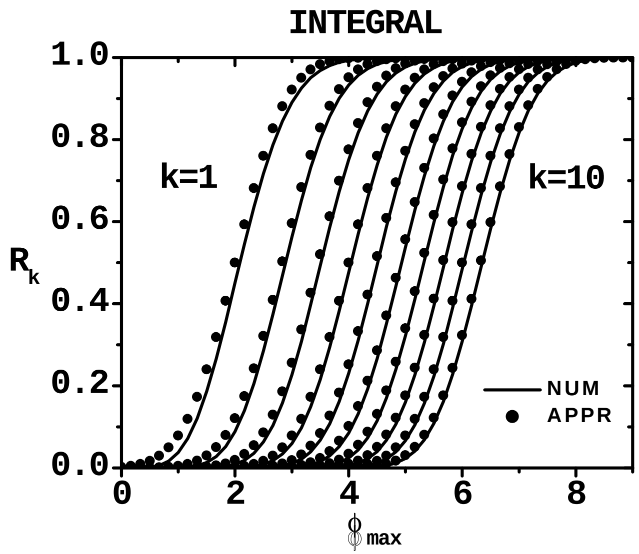

Figure 10: The numerical and fitting ratio of $R_k = \frac{a_k(\phi_{\max})}{a_k(\infty)}$ for k from 1 to 10.The line is accurate numerical result. The black circle is the fitting result.

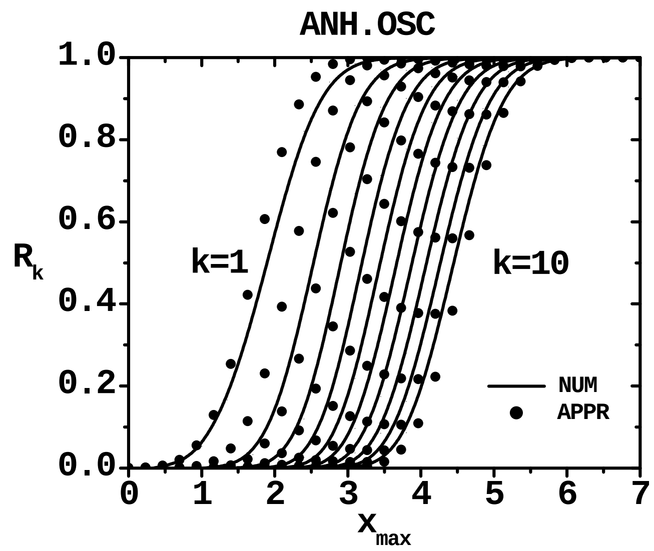

Figure 11: Numerical and approximate ratio $R_k = \frac{b_k(x_{\max})}{b_k(\infty)}$ versus $x_{\max}$. The line is the accurate numerical result. The black circles give the approximate formula for the anharmonic oscillator.