Department of Energy
Progress report Dec. 2007 - Nov. 2008

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Main Goals

- Development of improved perturbative methods constructed by removing large field configurations in the path integral and applicable for strong interactions.

- Understanding of the large order in perturbation for lattice QCD (no large field confgurations) in terms of the zeros of the partition function.

- Construction of lattice models with reduced finite size effects.

- Improvement of the local potential approximation in renormalization group equations.

Recent Results

- Models of the large order behavior of the perturbative expansion of the average plaquette (the lattice analog of the gluon condensate) in quenched QCD. The apparent singularities are off the real axis (in the complex $\beta = 2N/g^2$ plane; no third-order phase transition). Mean field theory explains the series up to order 20. Infrared renormalons probably dominate beyond order 25. Investigation of the zeros of the partition function for $U(1)$, $SU(2)$ and $SU(3)$ in progress.

- Semi-classical parametrizations of the non-perturbative part of the plaquette and the β-function in quenched QCD (corrections scale like the square of the lattice spacing). This suggests a semi-classical approach of the lattice scaling (to be developed).

- Universal behavior of the modified perturbative coefficients (calculated numerically with a large field cutoff) in quantum mechanics and for the hierarchical model. The computing time necessary to calculate the perturbative coefficients (with numerical, non-diagrammatic methods) scales like the order (instead of the factorial of the order for diagrammatic methods).

- Approximate equivalence between the renormalization group transformation of the hierarchical model and Polchinski's renormalization group equation in the local potential approximation (exponents differ by less than 10^{-5}). Calculations of the exponents for transformations where the volume is rescaled by integer values. Continuum limit in progress.

- Numerical calculation of the density of states ("color entropy") in lattice gauge theory. Smoother moments with reduced errors and good agreement with direct MC calculations.

- New methods to locate zeros of the partition function of $U(1)$ and $SU(2)$ gauge theories based on interpolations and polynomial approximation of the color entropy.

- Prescription to control nonlinear effects in Finite Size Scaling (FSS) (shrinking β interval procedure).

- Approximate equality between the value of β where the double peak plaquette distribution become symmetric and the real part of the leading zero of the partition function in the $U(1)$ case

Recent Publications

1. A. Denbleyker, D. Du, Y. Meurice, and A. Velytsky, *Fisher's zeros of quasi-Gaussian densities of states*, Phys. Rev. **D76** 116002 (2007), [arXiv:0708.0438 [hep-lat]].

2. A. Denbleyker, D. Du, Y. Meurice, and A. Velytsky, *Fisher's zeros and perturbative series in gluodynamics*, PoS **LAT2007**269 (2007) [arXiv:0710.5771 [hep-lat]].

3. Y. Meurice , *QCD at complex coupling, large order in perturbation theory and the gluon condensate*, to appear in Continuous Advances in QCD 2008 (World Scientific), e-Print: arXiv:0808.2458 [hep-th].

4. A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, and A. Velytsky, *Series expansions of the density of states in SU(2) lattice gauge theory*, Phys. Rev.**D78** 054503 (2008), e-Print: arXiv:0807.0185 [hep-lat].

5. A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, A. Velytsky *Approximate forms of the density of states*, PoS(LATTICE 2008)249, e-Print: arXiv:0810.2252 [hep-lat].

6. A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, A. Velytsky . *Volume dependence of Fisher's zeros* PoS(LATTICE 2008)244, e-Print: arXiv:0810.1792 [hep-lat].

7. Y. Meurice, *How to control nonlinear effects in Binder cumulants*, e-Print: arXiv:0712.1190 [hep-lat], expanded version in progress.

8. A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, and A. Velytsky , *Zeros of the partition function of* $SU(2)$ *lattice gauge theory*, preprint in progress

9. A. Bazavov, Y. Meurice, and A. Velytsky, *Density of states of* $U(1)$ *lattice gauge theory*, preprint in progress.

Recent Talks:

- Y. Meurice, Large order, large fields and finite T from lattice QCD at complex coupling, talk at Miami 2007, December 2007.
- Y. Meurice, QCD at complex coupling, large order in perturbation theory and the gluon condensate, talk at CAQCD08, Minneapolis , May 2008
- Y. Meurice, Linear and Nonlinear Aspects of Finite Size Scaling, talk at ERG2008, Heidelberg , July 2008
- Y. Meurice, Series expansions of the density of states in SU(2) lattice gauge theory, poster presented at ERG2008.
- Y. Meurice, Approximate forms of the density of states in pure gauge, talk at Lattice 2008, Williamsburg, July 2008.

- Y. Meurice, Density of States and Fisher Zeros in Lattice Gauge Theory, talk given at 2008 Midwest Theory Get-Together, October 2008.

- A. Denbleyker and Yuzhi Liu, Volume dependence of Fisher's zeros, poster presented at LATTICE 2008.

Proposal for a KITP program on the Renormalization Group

Last year, I wrote a solicited review article for Journal of Physics A "Nonlinear Aspects of the Renormalization Group Flows of Dyson's Hierarchical Model".

The article summarizes recent progress in doing numerical renormalization group calculations for scalar models. One of the merit of this review article was to start a dialogue with other authors using different methods where the renormalization group transformation evolves continuously instead of discretely as in our method.

This lead me and a few other people to submit a pre-proposal for a KITP program at UC Santa Barbara on the renormalization group. The director (David Gross) encouraged us to develop it. We are about ready to submit it (the deadline is December 15).

Graduate Student Supervision

I currently supervise four graduated students.

- Daping Du came in fall 2005. He has passed the qualifying exam and the comprehensive exam. He has passed the T.A. (Teaching Assistant) certification necessary to lead lab sections. He works on the fits of plaquette distribution, saddle point estimates of the Fisher zeros and the density of states in quenched QCD. He is now supported partially as a T.A. during the academic year and as a R.A. (Research Assistant) during summer.

- Alan Denbleyker came in fall 2006. He works on MC simulations in $SU(2)$ gauge theories with and without adjoint terms and is planning to extend the existing codes for $SU(3)$. He has devised complex zeros searching algorithm and has calculated the density of state numerically. Ha has started a study of Binder cumulants in finite temperature $SU(2)$

He is the system manager for our cluster and repository. He is supported as a T.A. during the academic year and as a R.A. during summer.

- Yuzhi Liu came in fall 2006. He has passed the qualifying exam. He has passed the T.A. certification. He works on the comparison between discrete renormalization group methods that we have been using and countinuous limits of these methods used by other authors. He has been supported partially as a T.A. and partially as a R.A..

- Zou Huyian came in fall 2008. He is now supported as a TA but has not passed the T. A. certification. He is taking an advanced class in quantum field theory during his first semester here and is attending our weekly meetings.

Computational Facilities

2003: we built a 16 node cluster that has been phased out

2006: we built and operated a new cluster with 8 single CPU nodes with 3.2 GHz Pentium 4 processors and Gigabyte motherboards with a build-in fast ethernet card.

We would like to upgrade in 2009. The company SiCortex, offers a 72-processor deskside that uses less than 300 Watts of power (for the entire deskside!) for $15,000 . The very low cost of operation and the ability to keep the deskside in a room with no special AC makes it an attractive way to solve the overcrowding of our departmental cluster room and it is very likely that we would get overhead return for half of the cost in order to favor this cost saving initiative.

We use Fermilab cluster (Proposal of type C accepted)

Work in progress

The density of states and color entropy

The partition function for a $SU(2)$ gauge theory, $Z(\beta)$, is the Laplace transform of $n(S)$, the density of states:

$$Z(\beta) = \int_0^{2\mathcal{N}_p} dS\ n(S)\ \mathrm{e}^{-\beta S}\ ,$$

where $\mathcal{N}_p = 6 \times L^4$ is the number of plaquettes. We define the color entropy

$$f(x, \mathcal{N}_p) \equiv ln(n(x\mathcal{N}_p, \mathcal{N}_p))/\mathcal{N}_p$$

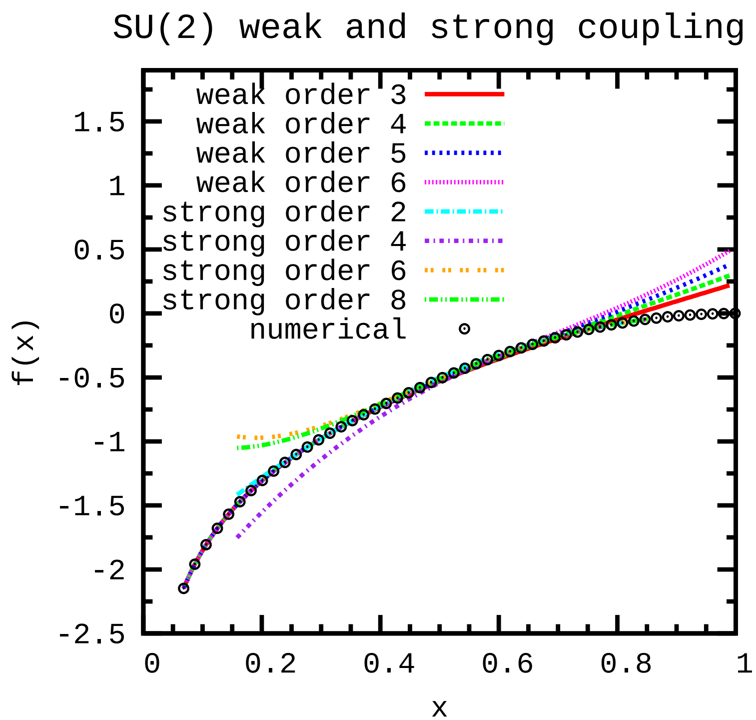

Figure 1: Weak and strong coupling expansion of f at a few intermediate orders compared with numerical data (notice nice overlap in central region).

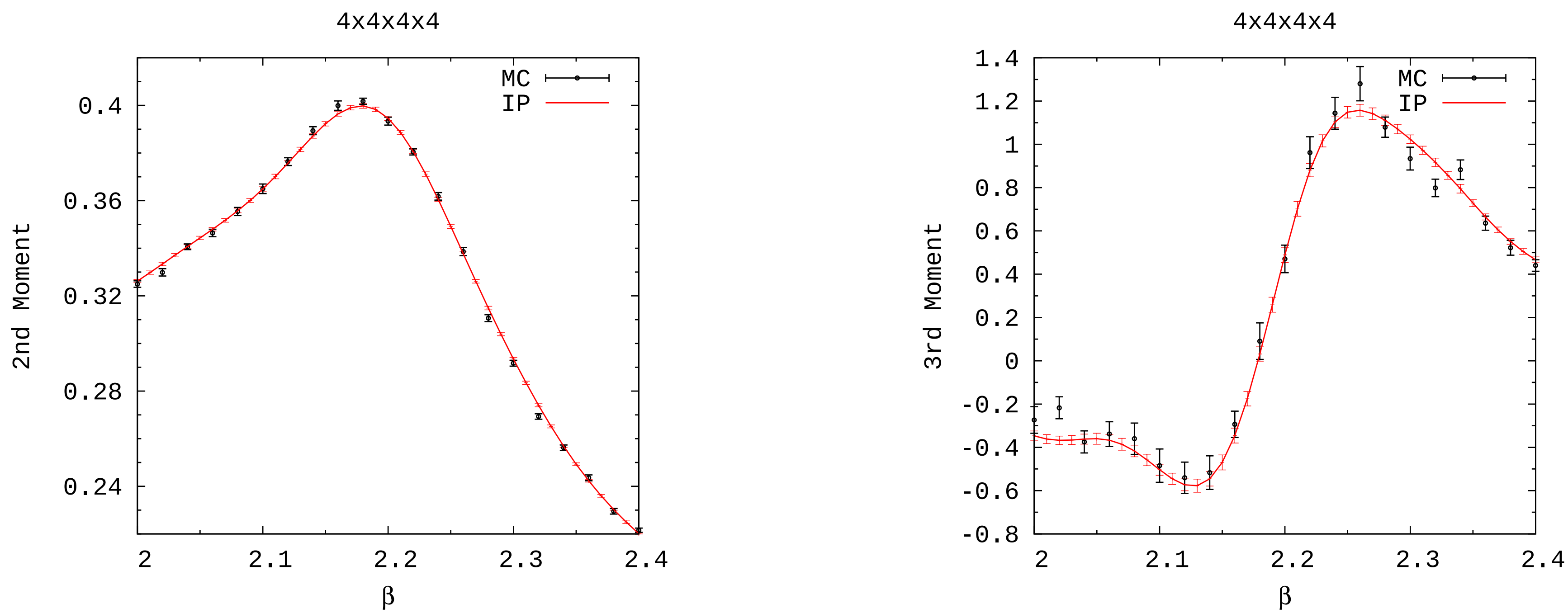

Figure 2: Comparison of the second and third moment calculated from the density of states and the direct MC result (Alan Denbleyker).

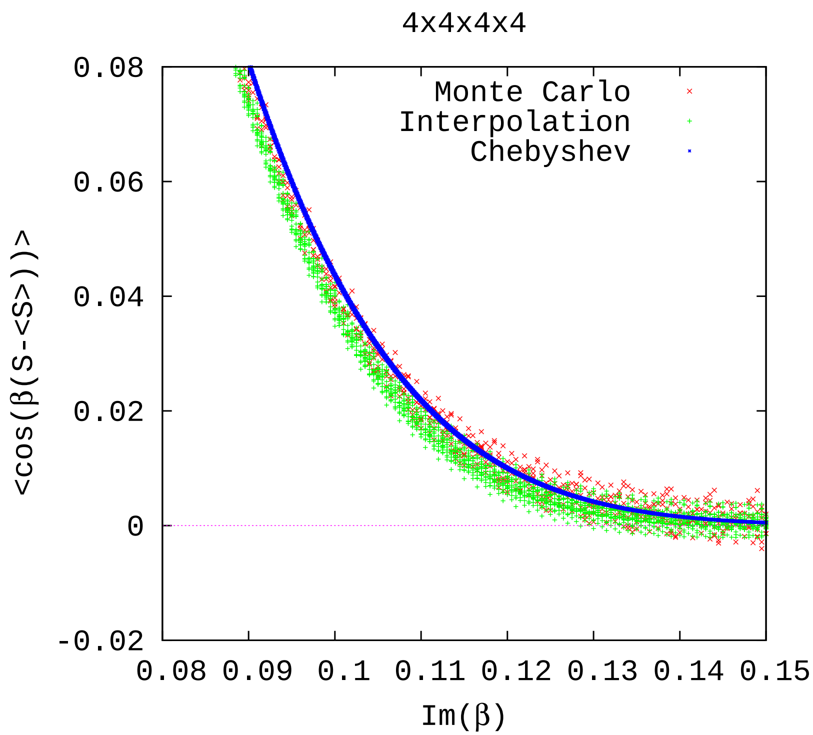

Figure 3: $< \cos(Im\beta(S- < S >)) >$ as a function of the imaginary part of β at fixed real part 2.18 with three methods: spline interpolation, Chebyshev fitting and Monte Carlo on a 4^4 lattice. The different sets are obtained by bootstraps.

Perturbative series in lattice gauge theory and Fisher zeros

Mean field assumption with singularity in the complex plane:

$$-\partial P/\partial\beta \propto \ln((1/\beta_m - 1/\beta)^2 + \Gamma^2) \ ,$$

Fisher zeros should stabilize at a distance $\Gamma\beta_m^2$ from the real axis when the volume increases.

Large order behavior consistent with $P(\beta) - P_{pert.}(\beta) \simeq C(a/r_0)^4$, with $a(\beta)$ defined with the force scale with $r_0 = 0.5$ fm.

C related to the so-called gluon condensate. The present calculation gives values 3-5 times larger than the values used in the continuum for phenomelogical purpose.

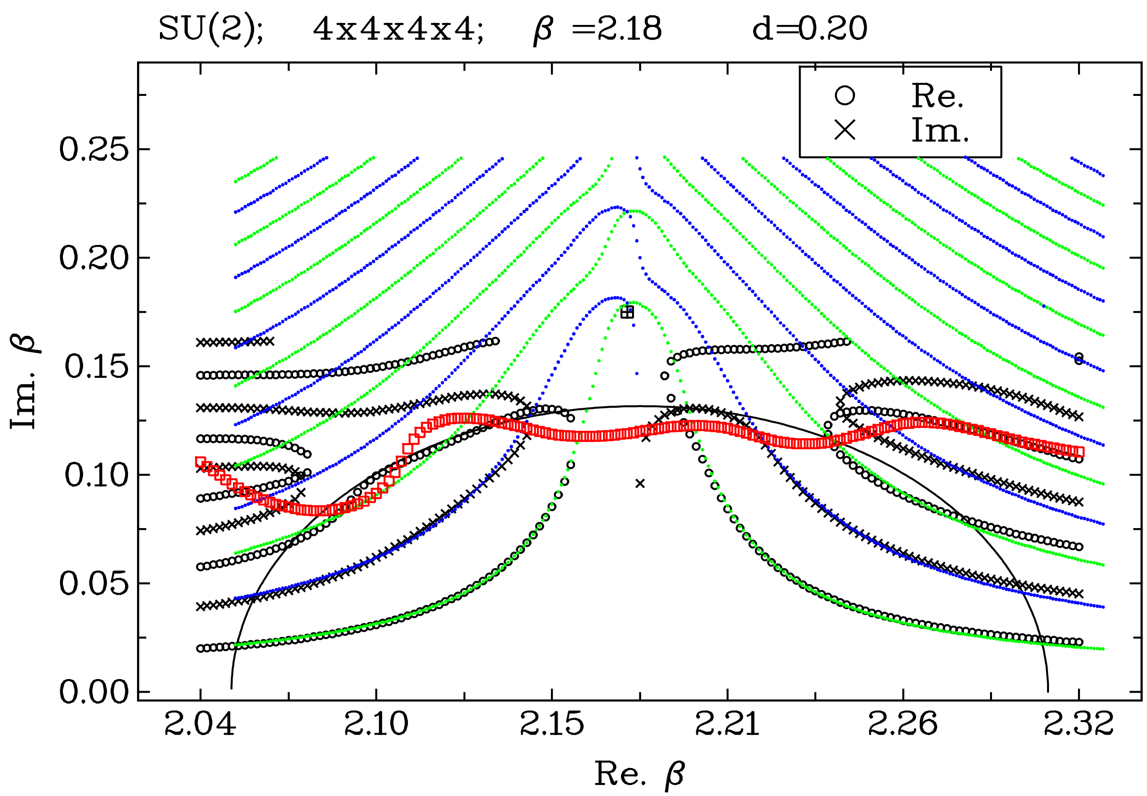

Figure 4: Zeros of the real (crosses) and imaginary (circles) using MC on a 4^4 lattice, for $SU(2)$ at $\beta_0 = 2.18$. The values for the real (green) and imaginary (blue) parts are obtained from a 4 parameter model.

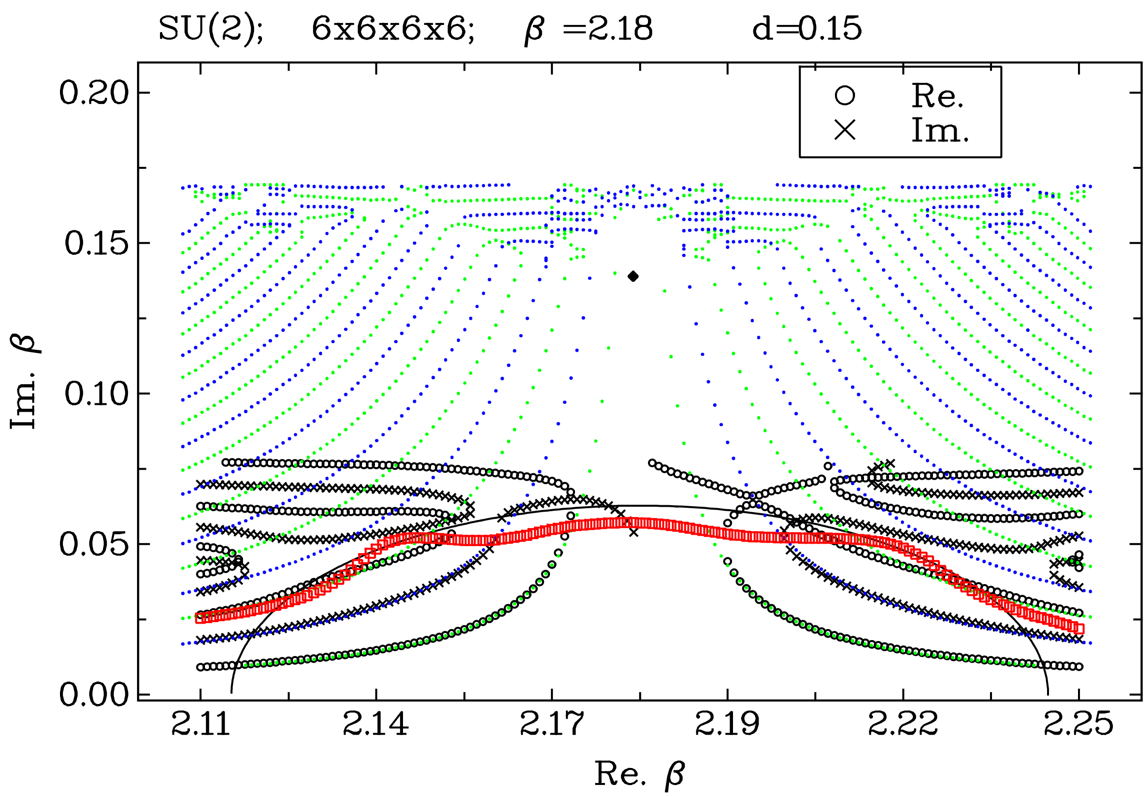

Figure 5: Same for a 6^4 lattice. The region of confidence for MC shrinks like $V^{-1/2}$.

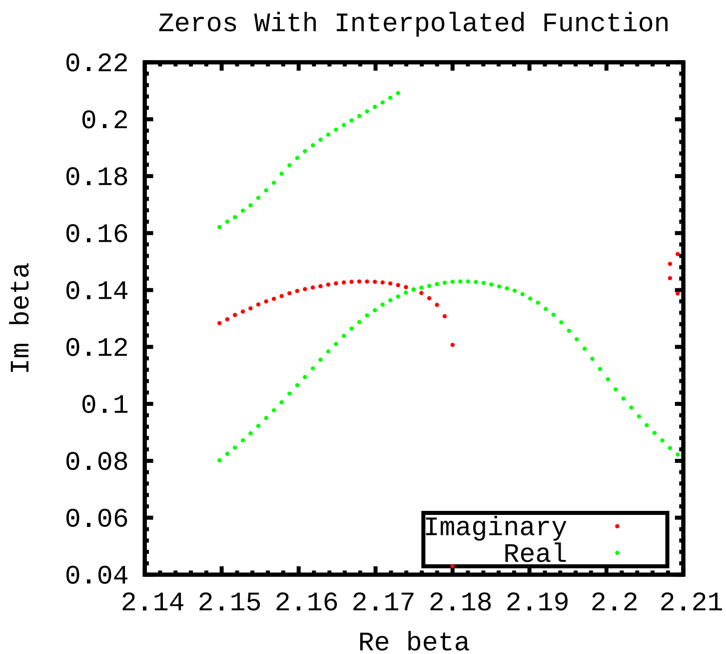

Figure 6: Same quantities on a 4^4 lattice but with an interpolated version of f.

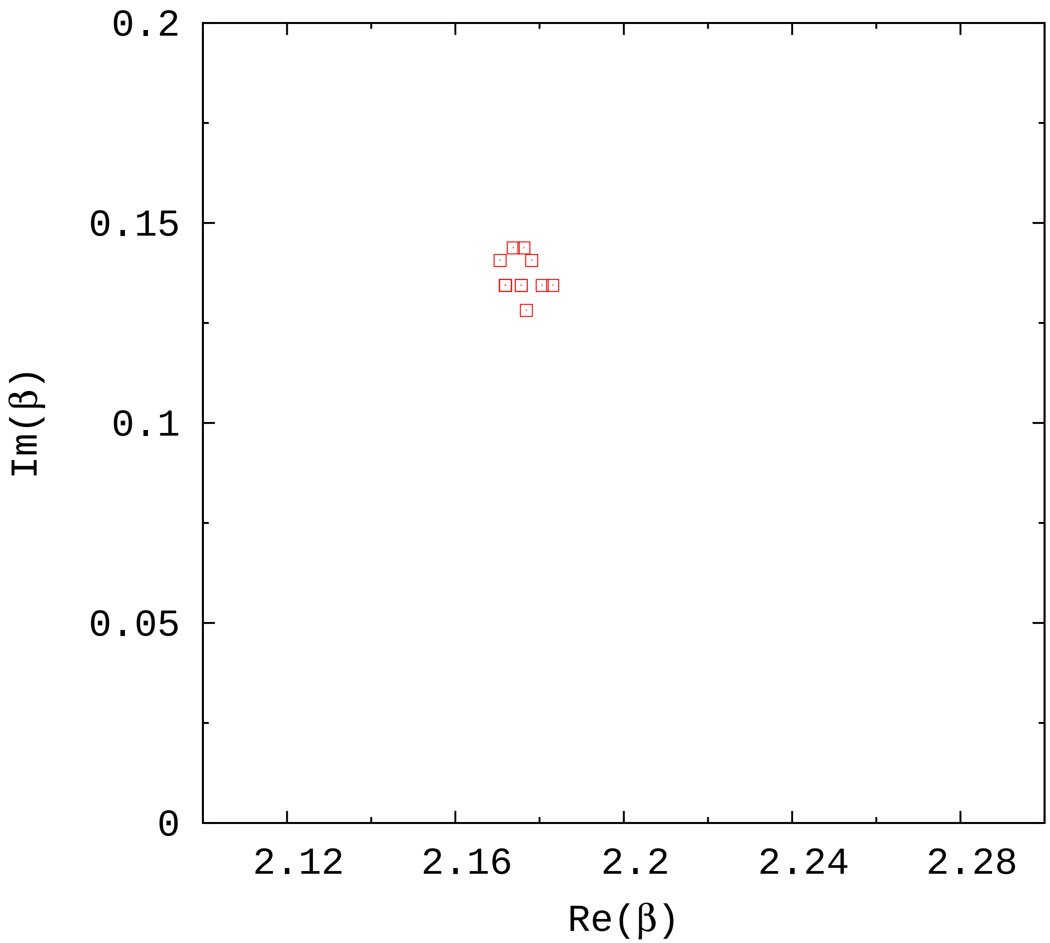

Figure 7: Spread of zeros with interpolated f and a new residue method (Daping Du)

$U(1)$ lattice gauge theory

Data obtained by multicanonical methods by A. Bazavov (Arizona) on a 4^4 lattice. A proposal of type C has been approved by Fermilab to pursue this study on larger lattices.

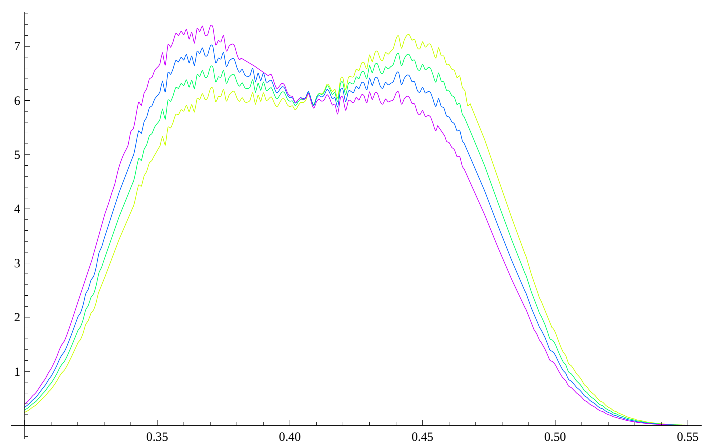

Figure 8: Plaquette distribution for $U(1)$ at β=0.978 (olive) , 0.979 (green, symm.), 0.98, and 0.981 (purple), using the density of states for a 4^4 lattice.

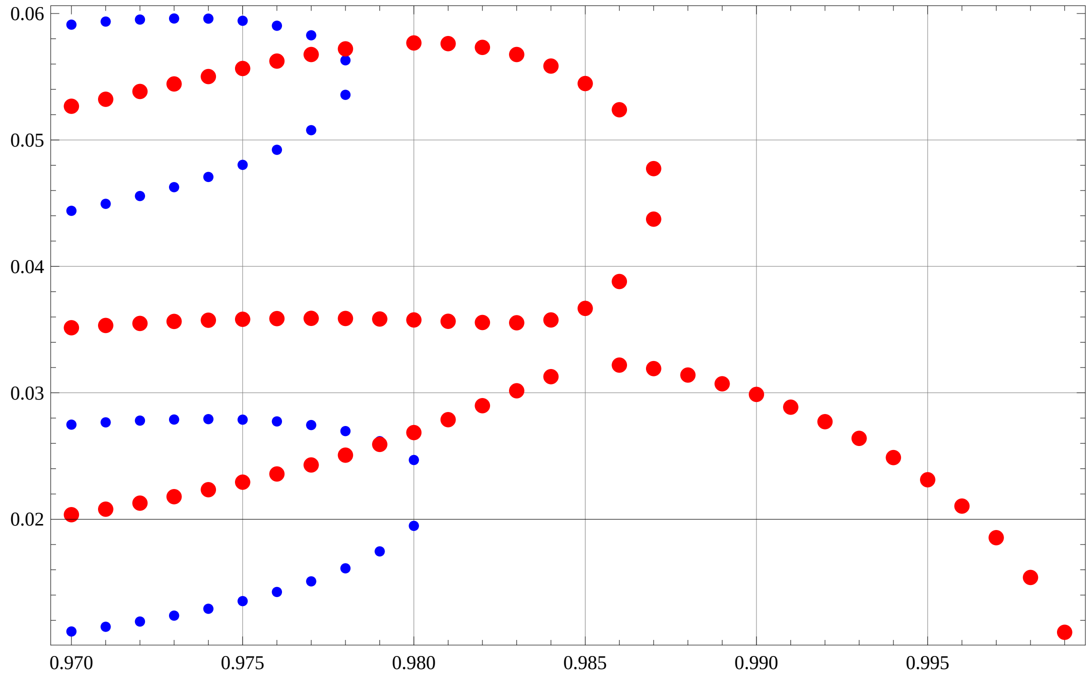

Figure 9: Zeros of Re and Im part of Z for $U(1)$ using the density of states for a 4^4 lattice. Real part of leading zero is about 0.979.

Recent work on finite size scaling

fourth Binder cumulant: $B_4 \equiv \frac{<m^4>}{<m^2>^2} = f(u_\kappa N^{1/\nu}, u_1 N^{-\omega_1}, u_2 N^{-\omega_2}, \ldots)$

$$B_4(\beta, N) \simeq B_4(\beta_c, \infty) + f_1 \kappa N^{1/\nu} + f_2 \kappa^2 N^{2/\nu} + (c_0 + c_1 \kappa N^{1/\nu}) N^{-\omega} \ .$$

N is the linear size The shrinking interval procedure.: In the literature, B_4 is often plotted for different volumes but at fixed values of β. It is better to shrink the interval as the volume increases.

$$|\beta - \bar{\beta}_c| < \epsilon (f_1/f_2) \bar{\beta}_c N^{-1/\nu} \ .$$

Remarkable crossings

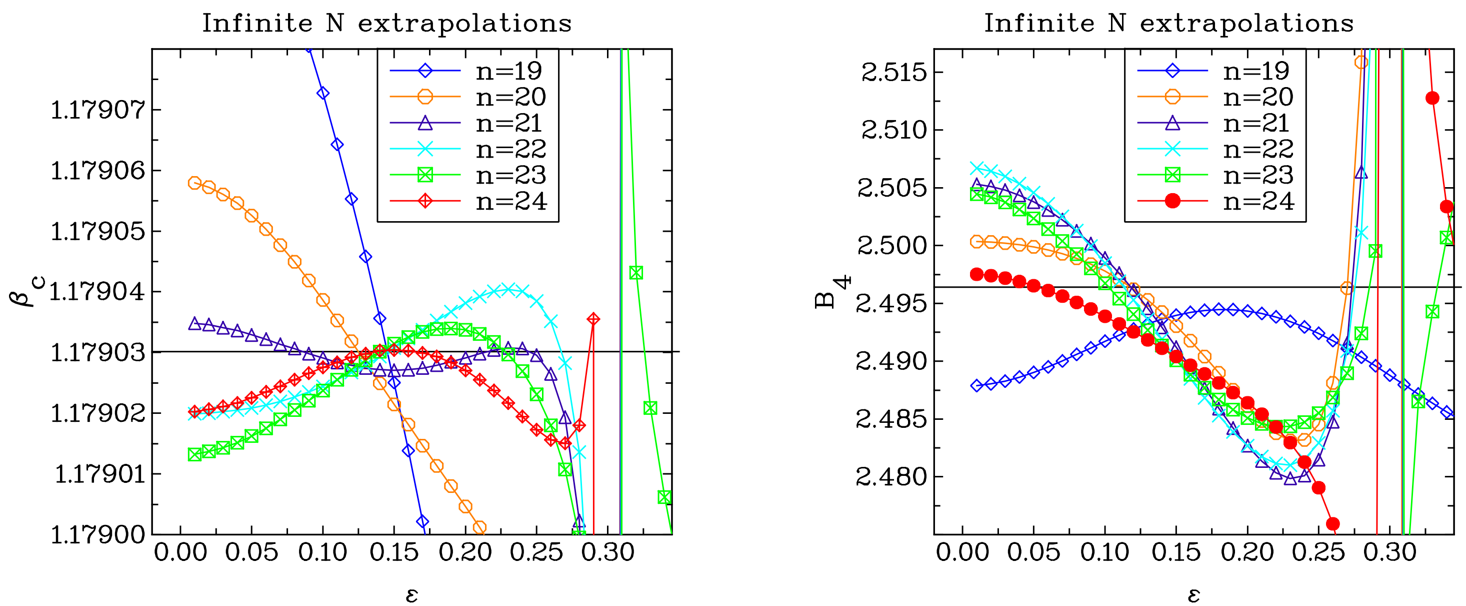

Figure 10: Infinite volume extrapolations of β_c and B_4 as a function of the optimization parameter ϵ.

B_4 for Polyakov loop for $SU(2)$ (with A. Velytsky)

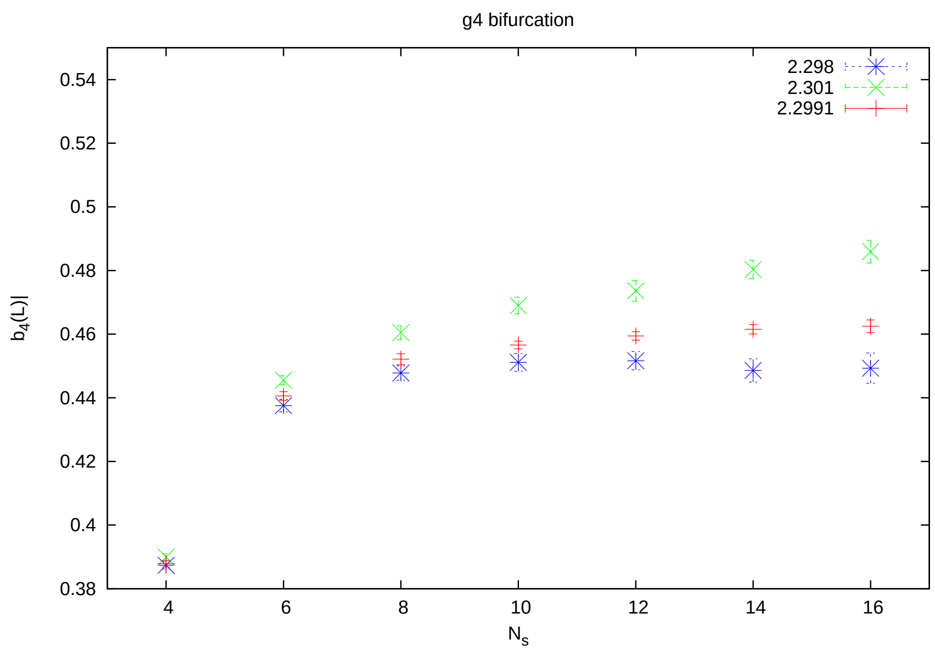

Figure 11: bifurcation of B_4 near β_c for $4 \times N_s^3$ lattice

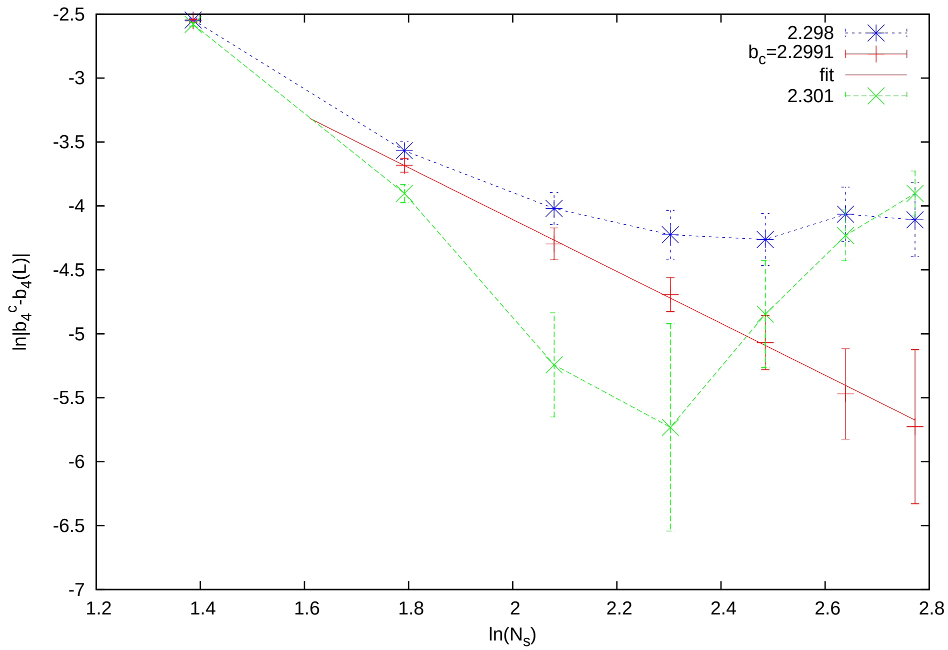

Figure 12: Finite size scaling near β_c. The slope is twice larger than expected!

The zero volume limit

$$B_4 = f(u_\kappa N^{1/\nu}, u_1 N^{-\omega_1}, u_2 N^{-\omega_2}, \dots)$$

The ω_i are widely spaced for the HM

$$\begin{aligned} \omega_1 &= 0.655736 \\ \omega_2 &= 3.17995 \\ \omega_3 &= 5.91212 \end{aligned}$$

A strategy to get accurate estimates at not too large volume is to try to fine tune u_κ and u_1 to the smallest possible values. Fine tuning u_1 can be done by looking for the crossing of the first and second irrelevant directions at very small volume. This was done for a LG measure.

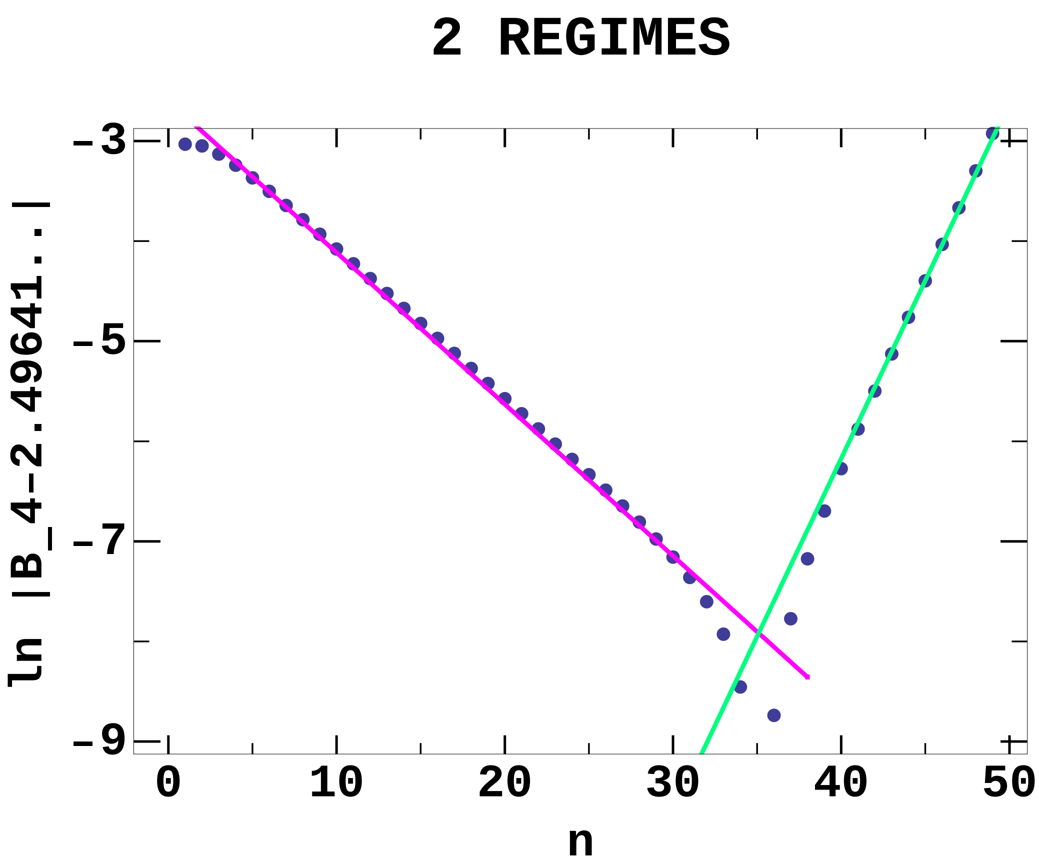

Figure 13: $\ln|B_4 - 2.49641845|$, versus $n = \text{Log}_2 V$. The two lines have slopes corresponding to the first irrelevant direction and the relevant direction (from left to right). β was fine tuned with 8 digits.

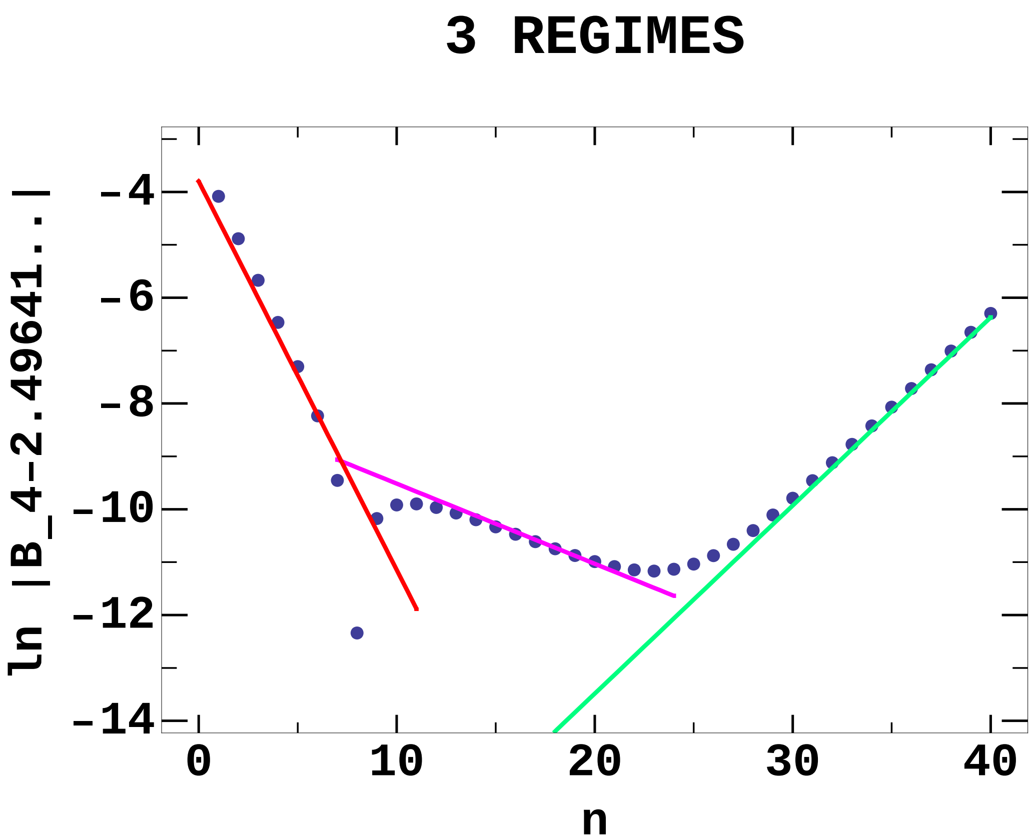

Figure 14: ln$|B_4 - 2.49641845|$, versus $n =$Log$_2 V$. The three lines have slopes corresponding to the second and first irrelevant directions and the relevant direction (from left to right). β was fine tuned with 8 digits and λ_4 with 3 digits.

Continuum limit of discrete RG

The recursion formula can be extended for arbitrary scale. The number of sites integrated for the HM, namely 2, appears as the exponent. With the replacements $2 \to \ell^D$ and $\frac{c}{4} \to \ell^{-2-D}$ the recursion formula becomes

$$R_{n+1}(k) = C_{n+1}\ \mathrm{e}^{-\frac{1}{2}\beta\frac{\partial^2}{\partial k^2}} \left(R_n(\sqrt{c/4}\ k)\right)^2 ,$$

becomes

$$R_{n+1}(k) = C_{n+1}\ \mathrm{e}^{-\frac{1}{2}\beta\frac{\partial^2}{\partial k^2}} \left(R_n(\ell^{-(D+2)/2}\ k)\right)^{\ell^D} ,$$

The usual equation is obtained for $\ell = 2^{1/3}$.

We are interested in the limit $\ell \to 1$. Working with the integral form, we get for V (essentially the log of R, see review)

$$\frac{\partial V}{\partial t} = DV + (1 - \frac{D}{2})\phi\frac{\partial V}{\partial \phi} - (\frac{\partial V}{\partial \phi})^2 + \frac{\partial^2 V}{\partial \phi^2} \tag{1}$$

which implies the so-called Wilson-Polchinski equation

$\nu_{HM} = 0.649570365$

$\nu_{WP} = 0.649561773$ (Litim; Bervillier, Juttner and Litim)

$\nu_{optimal} = \nu_{WP}$ (Litim)

Numerical issues

R^2 is a very simple multiplication of polynomials (when we use the polynomial truncation)

When ℓ^D is not integer, R^{ℓ^D} needs to be defined by some approximation.

We can use

$$
\begin{aligned}
R^{\ell^D} &= (1+(R-1))^{\ell^D} \\
&\simeq 1+\ell^D(R-1)+(1/2!)\ell^D(\ell^D-1)(R-1)^2+\ldots
\end{aligned}
$$

As $R-1$ is of order k^2, it is consistent to truncate the sum at order $(R-1)^{l_{max}}$. Usual polynomial approximations do not seem to converge.

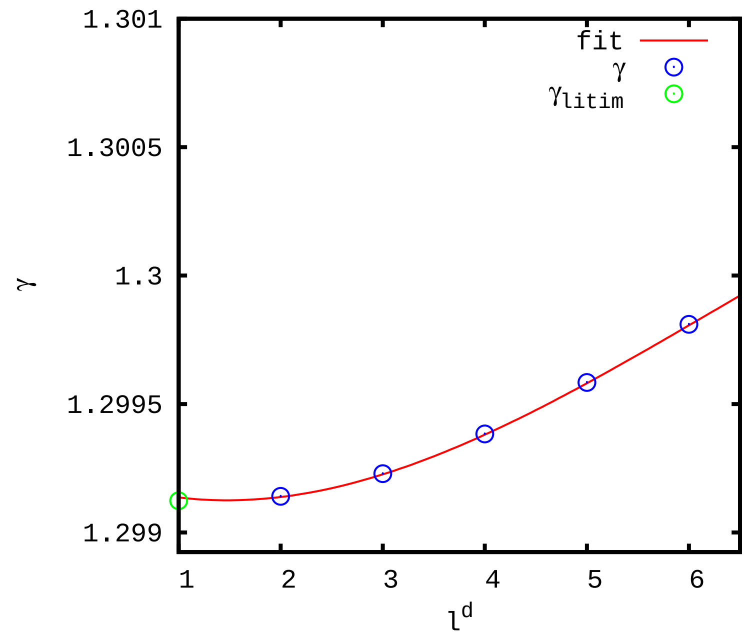

Figure 15: critical exponents that are numerically stable when the number of sites blocked is integer. We are working on a perturbative interpolation method (Liu Yuzhi) .