Department of Energy
Progress report Fall 2006 -Fall 2009

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Main Interests

Main interest: models of strong interactions and lattice models

Applications: QCD and extensions beyond the standard model

Methods: improved perturbation theory and renormalization group methods

Computational facilities: Linux clusters

New computational possibilities being explored: optical lattice realizations of lattice gauge theory models (if it turns out to be feasible)

Main Goals

- Development of improved perturbative methods constructed by removing large field configurations in the path integral and applicable for strong interactions.

- Understanding of the large order in perturbation for lattice QCD (no large field confgurations) in terms of the zeros of the partition function.

- Construction of lattice models with reduced finite size effects.

- Improvement of the local potential approximation in renormalization group equations.

Recent Results

- Models of the large order behavior of the perturbative expansion of the average plaquette (the lattice analog of the gluon condensate) in quenched QCD. The apparent singularities are slightly off the real axis (in the complex $\beta = 2N/g^2$ plane; no third-order phase transition).The mean field theory model explains the series up to order 30 (Perlt et al. Lattice 2009). Infrared renormalons could dominate beyond that order.

- Numerical calculation of the density of states (”color entropy”) in in $SU(2)$ and $U(1)$ (with A. Bazavov) lattice gauge theory. Check of the moments with direct MC calculations.

- New methods to locate the zeros of the partition function (Fisher’s zeros) of $U(1)$ and $SU(2)$ gauge theories. Evidence that the lowest zero stabilizes as the volume increases for $SU(2)$. Relation between symmetric double peak β and real part of zeros of the partition function in the $U(1)$ case.

- Modification of Dyson's instability argument for lattice gauge theory with compact groups and $O(N)$ nonlinear sigma models. Modified dispersion relations. The zeros of the β function in the complex plane can be seen as singular points of the mapping between the coupling and the mass gap. Arguments for stabilization of Fisher's zeros above the real axis in the large-N limit. Explicit checks at finite N and volume.

- Method to control nonlinear effects in Finite Size Scaling (FSS). Improvement of the accuracy of the critical exponents for the finite temperature transition of $SU(2)$. Corrections to scaling seem dominated by anisotropic effects ($\omega \simeq 2$).

- Approximate equivalence between the renormalization group transformation of the hierarchical model and Polchinski's renormalization group equation in the local potential approximation (exponents differ by less than 10^{-5}). Attempt to interpolate the exponents between transformations where the volume is rescaled by integer values.

Recent Publications

- Y. Meurice, *The non-perturbative part of the plaquette in quenched QCD*, Phys. Rev. **D74**, 096005 (2006).

- A. Denbleyker, D. Du, Y. Meurice, and M. Naides, *Definition and parametrization of non-perturbative effects in quenched QCD*, PoS **LAT2006** (2006) 215.

- Y. Meurice, *At which order should we truncate perturbative series?, Minneapolis 2006, CAQCD* 310-316, World Scientific (2007).

- Y. Meurice, *Nonlinear Aspects of the Renormalization Group Flows of Dyson's Hierarchical Model*, J.Phys. **A40** R39-102 (2007) (Sollicited review article).

- A. Denbleyker, D. Du, Y. Meurice, and A. Velytsky, *Fisher's zeros of quasi-Gaussian densities of states* Phys. Rev. **D76** 116002 (2007).

- A. Denbleyker, D. Du, Y. Meurice, and A. Velytsky, *Fisher's zeros and perturbative series in gluodynamics*, PoS **LAT2007**269 (2007).

- Y. Meurice, *QCD at complex coupling, large order in perturbation theory and the gluon condensate*, to appear in CAQCD 2008 (World Scientific).

- A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, and A. Velytsky, *Series expansions of the density of states in SU(2) lattice gauge theory*, Phys. Rev. **D78** 054503 (2008).

- A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice , and A. Velytsky, *Approximate forms of the density of states* PoS(LATTICE 2008)249.

- A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, and A. Velytsky, *Volume dependence of Fisher's zeros* PoS(LATTICE 2008)244.

- Y. Meurice, *Dyson instability for 2D nonlinear O(N) sigma models*, Phys.Rev. **D80**, 054020 (2009).

- A. Denbleyker, Daping Du, Y. Meurice, and A. Velytsky, *Finite size scaling of Fisher's zeros for* $SU(2)$ *pure gauge theory*, e-Print: arXiv:0911.1831 [hep-lat].

- A. Bazavov, A. Denbleyker, Daping Du, Y. Meurice, A. Velytsky, Haiyuan Zou, *Dyson's Instability in Lattice Gauge Theory* e-Print: arXiv:0910.5785 [hep-lat]

Recent Talks:

ERG 2006 (Lefkada, Oct. 06), LoopFest VI (Fermilab, Apr. 07), U. Kansas (Lawrence, Apr. 07), LATTICE 2007 (Regensburg, Aug. 07), XQCD (Frascati, Aug. 07), Miami 2007 (Dec. 2007), CAQCD (Minneapolis, May 08), ERG 2008 (Heidelberg, July 08), LATTICE 2008 (Williamsburg, July 2008), Midwest Theory Get-Together, (Argonne, Oct. 08), Miami 2008, (Dec. 2008), Large-N workshop (INT Seattle, Feb. 09), CCNY (March 09), Joint Theory Workshop (Argonne, Apr. 09), Lattice 2009 (Beijing, July 09), Quantum Gauge Theories (San Benet, Sept 09), Midwest Theory Get-Together, (Argonne, Oct. 09).

Recent Talks by Students: D. Du and A. Denblyker (talks at the Summer School on Lattice QCD, Seattle, Aug. 2007), A. Denbleyker and Yuzhi Liu, (poster at LATTICE 2008), Yuzhi Liu (poster at LATTICE 2009 and talk at KITPC), Yuzhi Liu and Haiyuan Zou (talks at APS conference, Nov. 09)

Conference Organization

- Proposal for a KITP program at UC Santa Barbara on the renormalization group. The director (David Gross) encouraged us to develop it but it was not selected.

- *New Applications of the Renormalization Group Method*, INT workshop, Feb. 22-26 2010, with M. Birse, and S.-W.Tsai

- *Critical Behavior of Lattice Models*, Aspen Workshop, May 24 -June 11 2010, with G. Baym, U. Schollwoeck and S.-W. Tsai. Will involve optical lattice realizations.

Graduate Student Supervision

- Daping Du came in fall 2005. He has passed the qualifying exam and the comprehensive exam. He attended a Summer School In 2007 at UW Seattle. Works on localization of Fisher zeros using the density of states in $SU(2)$. Supported as a T.A. during the academic year and as a R.A. (Research Assistant) during summer. Expected graduation: May 2010.

- Alan Denbleyker came in fall 2006. He has passed the qualifying exam. He attended a Summer School In 2007 at UW Seattle. He works on MC simulations in $SU(2)$ gauge theories with and without adjoint terms. He calculates the density of state numerically and studies the Binder cumulants for finite temperature $SU(2)$. He is the system manager for our cluster and repository. He is supported as a T.A. during the academic year and as a R.A. during summer.

- Yuzhi Liu came in fall 2006. He has passed the qualifying exam. Attended summer schools at KITPC and Les Houches. He works on finite size scaling and the comparison between discrete renormalization group methods that we have been using and countinuous limits of these methods used by other authors. He has been supported partially as a T.A. and partially as a R.A.

- Haiyuan Zou came in fall 2008. He has passed the qualifying exam. He was supported as a TA but has not passed the T. A. certification yet. He is now supported as RA. He has been taking two advanced class in quantum field theory. He is working on the mapping between the mass gap and the coupling constant in the complex plane and the volume dependence of Fisher's zeros.

Computational Facilities

2003: we built a 16 node cluster that has been phased out

2006: we built and operated a new cluster with 8 single CPU nodes with 3.2 GHz Pentium 4 processors and Gigabyte motherboards with a build-in fast ethernet card.

We would like to upgrade the first cluster keeping the existing rack cabinets. We have build 4 nodes with quad core processors that are now running properly and cost $259/node. We would like to add 12 other nodes and upgrade the switch and UPS batteries.

One plaquette ($SU(2)$)

$Z(\beta) = \int_0^2 dS n(S) \mathrm{e}^{-\beta S} = 2\mathrm{e}^{-\beta} I_1(\beta)/\beta$ (analytical in the entire β plane)

$n(S) = \frac{2}{\pi}\sqrt{S(2-S)}$ (invariant under $S \rightarrow 2-S$)

The large order of the weak coupling expansion $\beta \rightarrow \infty$ is determined by the behavior of $n(S)$ near $S = 2$, itself probed when $\beta \rightarrow -\infty$ in agreement with the common wisdom that the large order behavior of weak coupling series can be understood in terms of the behavior at small negative coupling.

$\sqrt{2-S}$ is easy to approximate near $S = 0$ (radius of convergence $= 2$)

$Z(\beta) = (\beta\pi)^{-3/2} 2^{1/2} \sum_{l=0}^{\infty} (2\beta)^{-l} \frac{\Gamma(l+1/2)}{l!(1/2-l)} \int_0^{2\beta} dt \mathrm{e}^{-t} t^{l+1/2}$ is convergent

The crucial step

$\int_0^{2\beta} dt \mathrm{e}^{-t} t^{l+1/2} \simeq \int_0^{\infty} dt \mathrm{e}^{-t} t^{l+1/2} + O(\mathrm{e}^{-2\beta})$ is responsible for the factorial behavior

The peak of the integrand crosses the boundary near order 2β

Dropping higher order terms (than order $\simeq 2\beta$) agrees with the rule of thumb (minimizing the first contribution dropped)

The non-perturbative part can be fully reconstructed (higher orders + "tails", PRD 74 096005)

For L^4 lattices, the crossing is near order $2\beta\mathcal{N}_p$. Non-perturbative effects should be explainable by the contributions near S_{max} which can be probed at small negative coupling

Large order behavior of the average plaquette P

The simplest quantity for which the question of the large order behavior of lattice perturbation theory can be addressed is the weak coupling expansion of the average plaquette P. Recent numerical calculations of this expansion for a pure $SU(3)$ lattice gauge theory in 4 dimensions suggest a nonanalytical power behavior near $\beta = 6/g^2 \simeq 5.8$. Standard estimators for the power behavior indicate a singularity in the third derivative of the free energy. We have shown that the peak in this quantity present on 4^4 lattices disappears if the size of the lattice is increased isotropically up to a 10^4 lattice. This together with the absence of massless states for Wilson's action in the fundamental representation, can be resolved by moving the singularity slightly away from the real axis in the complex $1/\beta$ plane

We proposed a simple parametrization of the perturbative series based on the mean field assumption that $\partial P/\partial\beta$ has a logarithmic singularity in the complex β plane. After integration, we obtain the expansion

$$P \simeq A(\mathrm{Li}_2(\beta^{-1}/(\beta_m^{-1}+i\Gamma)) + \mathrm{h.c} = \sum_{k=1} A(\beta^{-1}/(\beta_m^{-1}+i\Gamma))^k/k^2 + \mathrm{h.c}\ ,$$

The parameters A and β_m can be determined with the numerical values of the two highest coefficients. For a series up to order 10, the effect of Γ is undistinguishable from the numerical errors as long as $\Gamma < 0.005$. Except for the first two coefficients, this method provides remarkably accurate predictions of the other coefficients. Series of order 20-30 are necessary to resolve Γ (see work in progress).

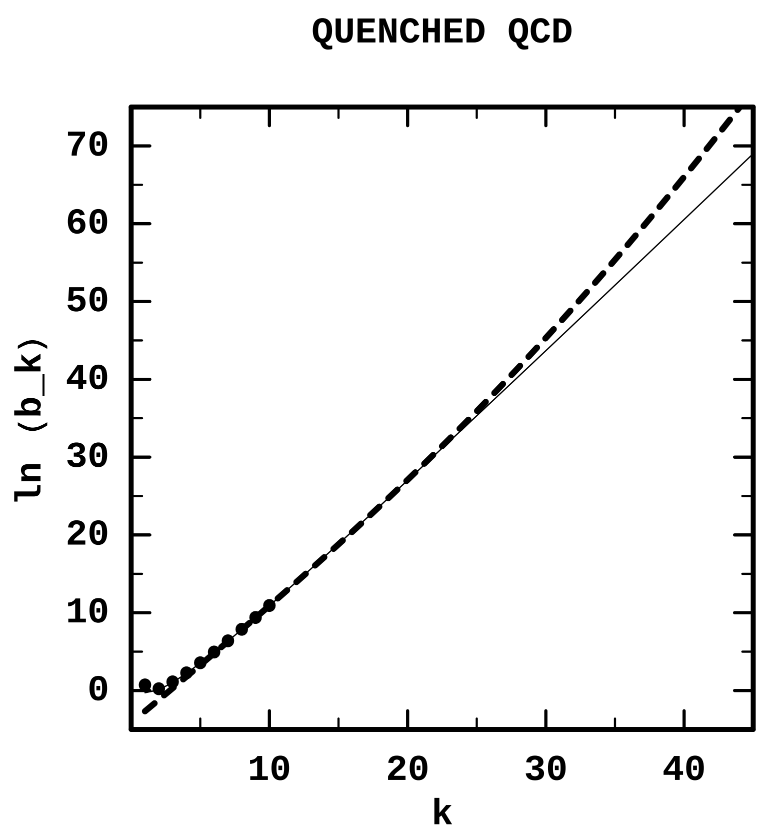

Figure 1: $\ln(b_k)$ for the dilogarithm model (solid line) and the integral model (dashes). The dots up to order 10 are the known values. The two models yields similar coefficients up to order 20. After that, the integral model has the logarithm of its coefficients growing faster than linear.

This model provides a large order extrapolation of the perturbative series. It is possible to compare accurate numerical values of P with the series at successive orders. This is illustrated in Fig. 2. The accumulation of error curves as the order increases is consistent with

$$P_{numerical}(\beta) - P_{perturbative}(\beta) \simeq C(a/r_0)^4$$

with the nonperturbative corrections to the running of the lattice spacing $a(\beta)$ defined with the so-called force scale with $r_0 = 0.5$ fm

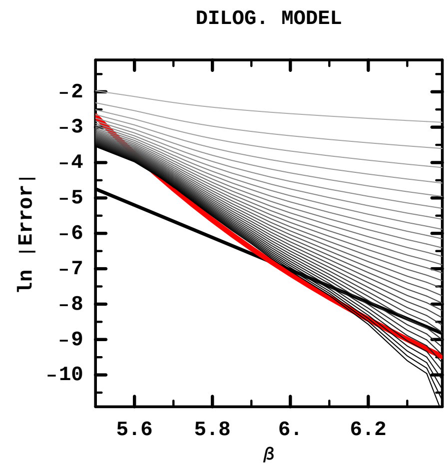

Figure 2: Natural logarithm of the absolute value of the difference between the series and the numerical value for order 1 to 30 for quenched QCD with the dilogarithm model. As the order increases, the curves get darker. The long dash curve is ln$(0.65\ (a/r_0)^4)$. The solid curve is the two loop perturbative result ln$(3.1 \times 10^8 \times (\beta)^{204/121-1/2} \mathrm{e}^{-(16\pi^2/33)\beta})$

Attempts have been made in the past to relate C to the so-called gluon condensate. The value that could in principle be compared with the commonly used value of 0.012 GeV^4 is $(36/\pi^2)Cr_0^{-4}$ for $N_c = 3$. C is sensitive to resummation. $C \simeq 0.6$ with the bare series and 0.4 with the tadpole improved series of P. Rakow. This gives values 3-5 times larger than the value quoted above. Beside the question of scheme dependence, the gluon condensate is not an order parameter and it seems difficult to compare the lattice results with quantities defined in the context of sum rules. I

Location of Fisher's Zeros

The singularities of P appear at the zeros of the partition function in the β plane called Fisher's zeros. They can be located by reweighting the plaquette distribution $P(S)$ generated with the MC method. The distribution of values of S is approximately Gaussian. This can be used to define a radius of confidence in the complex β plane that shrinks like $V^{-1/2}$. We have developed new methods to find zeros of the partition that lay outside of the region of confidence of MC calculations. First, we approximated the plaquette distribution at fixed β by the exponential of a polynomial of degree 4 (the results are illustrated in Fig. 3) and later we used the method of the density of state (see c. below).

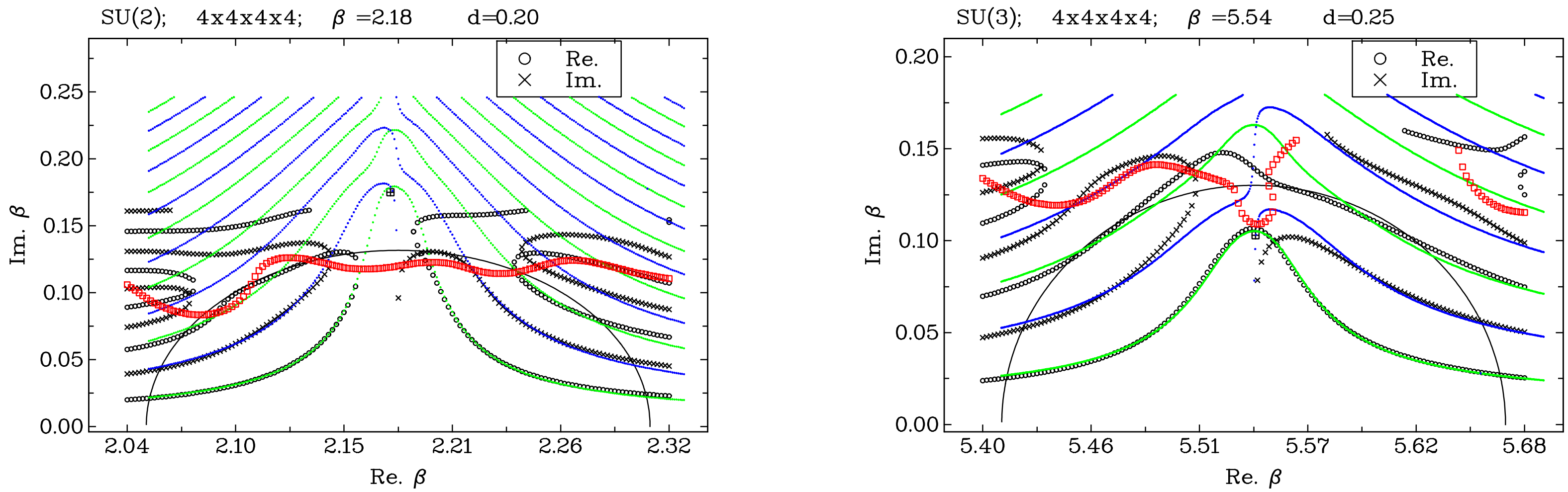

Figure 3: Zeros of the real (crosses) and imaginary (circles) using MC on a 4^4 lattice, for $SU(2)$ at $\beta =$ 2.18 and $SU(3)$ at $\beta =$5.54. The smaller dots are the values for the real (green) and imaginary (blue) parts obtained from the 4 parameter model. The MC exclusion region is represented by red boxes (see PRD 76 116002)

There exists a simple relation between the poles of the average plaquette and the zeros of the partition function. If β_0 is a zero of Z of order k, then $(dZ/d\beta)/Z \simeq k/(\beta - \beta_0)$ for $\beta \simeq \beta_0$. If we now integrate over a closed contour C,

$$(i2\pi)^{-1} \oint_C d\beta (dZ/d\beta)/Z = \sum_k n_k(C) \ , \qquad (1)$$

where $n_k(C)$ is the number of zeros of order k inside C. Z and its derivative were calculated using the density of state method explained below. The accuracy of the above contour integral can be monitored by checking that the real part is an integer (see left part of Fig. 4). This method was used by D. Du to locate the boundary of the region where Fisher zeros are present. (see right part of Fig. 4). These results were presented at Lattice 2009. Our preliminary results are consistent with a L^{-2} scaling for $Im\beta$ of the lowest zero.

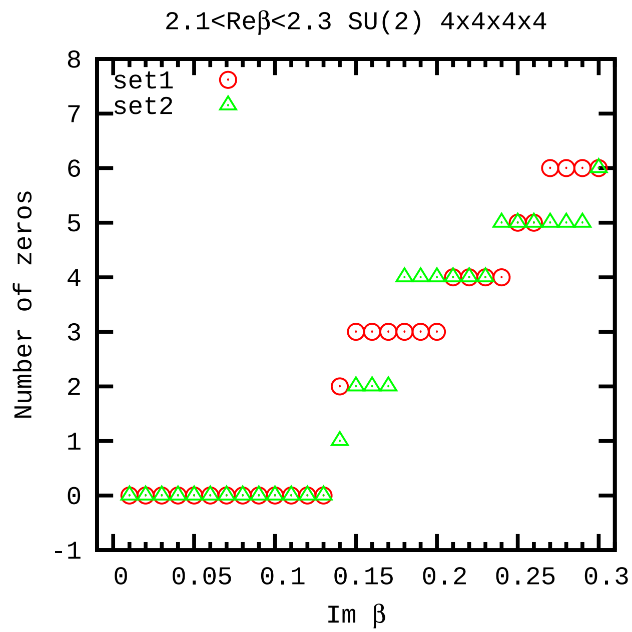

Figure 4: $Re\sum_k n_k$ for a rectangle based on the real axis with $2.1 < Re\beta < 2.3$ and a variable upper part for independent density of states.

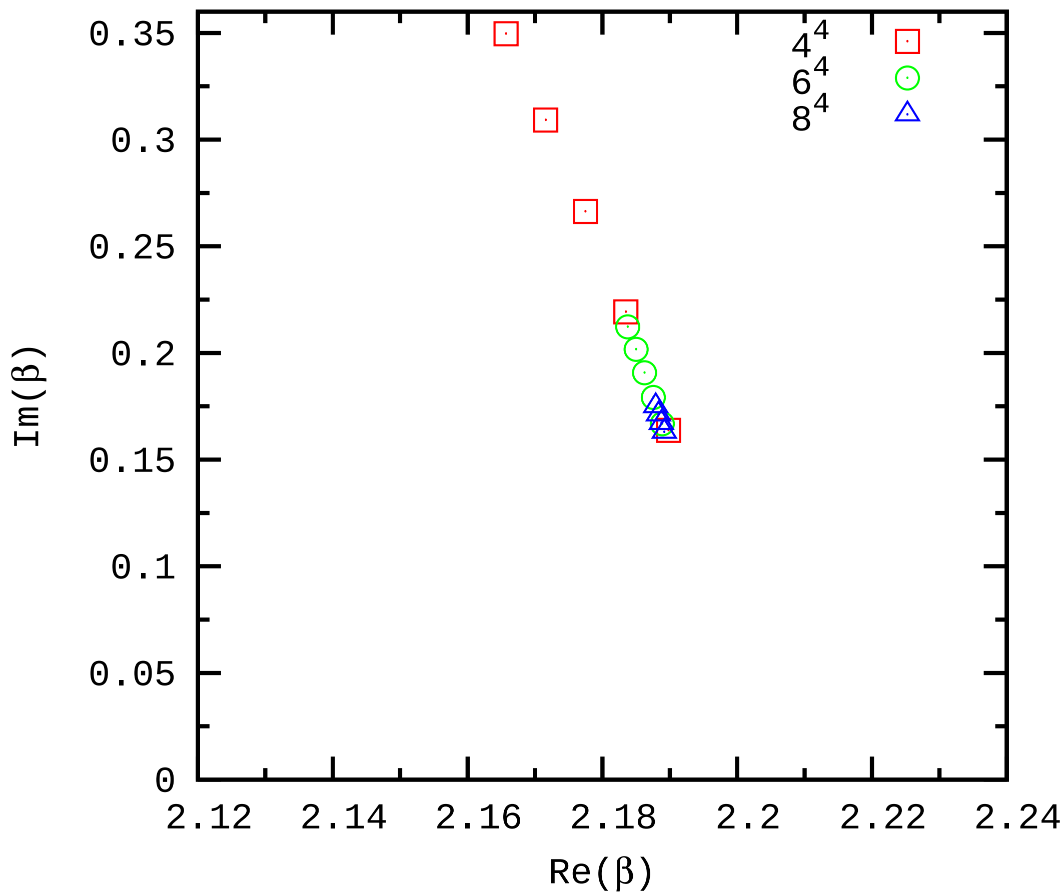

Figure 5: Fisher's zeros for $SU(2)$ on 4^4 , 6^4 and 8^4 (left). On the right, the distance between the zeros has been rescaled by $(4/L)^{3.7}$. Work done by Daping Du

integrand oscillate rapidly. A preliminary idea of the distribution of zero can be obtained using semi-classical methods. Using the “color entropy” $f(s)$ defined in Eq. (**??**), the saddle point of the integral is at s_0 given by solving $f'(s_0) = \beta$. Z becomes a Gaussian integral with correction of order $\sqrt{1/\mathcal{N}_p}$ as long as $Ref''(s_0) < 0$. As a Gaussian density of states has no complex zeros [**?**], it seems clear that zeros should appear in regions of the β plane corresponding to regions of the s plane such that $Ref''(s_0) > 0$. Using Chebyshev approximations of $f(s)$, we have constructed the boundary ($Ref''(s) = 0$). The results are shown in Fig. 6. The boundary form narrow tongues ending at a complex zeros of f''. These complex zeros are then mapped in the β plane using f'. Their number depends on the degree of the polynomial approximation, but the general shape is robust under changes in the degree. It appears that in the case of $SU(2)$ the images in the β plane are never on the real axis in contrast to the case of $U(1)$.

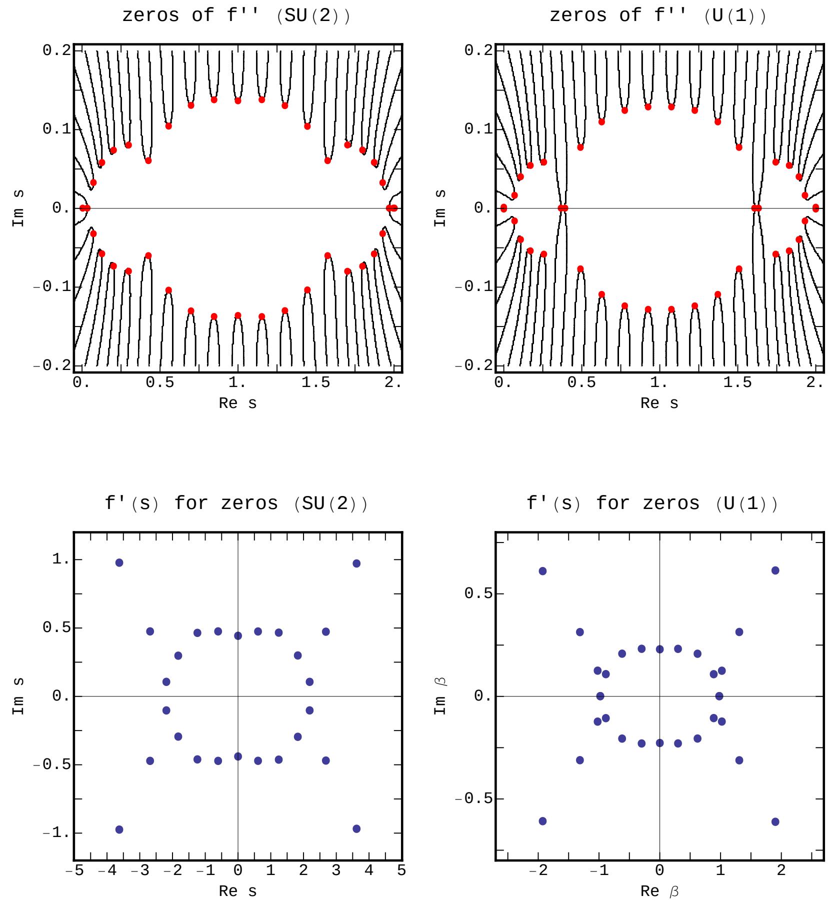

Figure 6: Top: complex zeros and zeros of the real part of $f''(s)$ in the complex s plane with 40 Chebyshev polynomials on 4^4 for $SU(2)$ (left) and $U(1)$ (right). Bottom: $f'(s)$ evaluated at the complex zeros of $f''(s)$ shown on the previous figure for $SU(2)$ (left) and $U(1)$ (right).

The density of states and color entropy

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The partition function for a $SU(2)$ gauge theory, $Z(\beta)$, is the Laplace transform of $n(S)$, the density of states:

$$Z(\beta) = \int_0^{2\mathcal{N}_p} dS \; n(S) \; \mathrm{e}^{-\beta S} \ , \tag{2}$$

where $\mathcal{N}_p = 6 \times L^4$ is the number of plaquettes. We define the color entropy $f(x, \mathcal{N}_p) \equiv ln(n(x\mathcal{N}_p, \mathcal{N}_p))/\mathcal{N}_p$. A. Denbleyker calculated f numerically on L^4 lattices. Small volume dependence were resolved for small values of S. We compared f with weak and strong coupling expansions. Intermediate order expansions show a good overlap for values of

S corresponding to the crossover (see Fig. 7 , left). We were able to relate the convergence of these expansions to those of the average plaquette. When known logarithmic singularities are subtracted from f, expansions in Legendre polynomials appear to converge uniformly. Subsequently, we found that discrete Chebyshev approximations of f were the most stable under numerical fluctuations. This is illustrated in Fig. 7 (right). This method is being used to find zeros at larger values of the imaginary part of β as explained in b. .

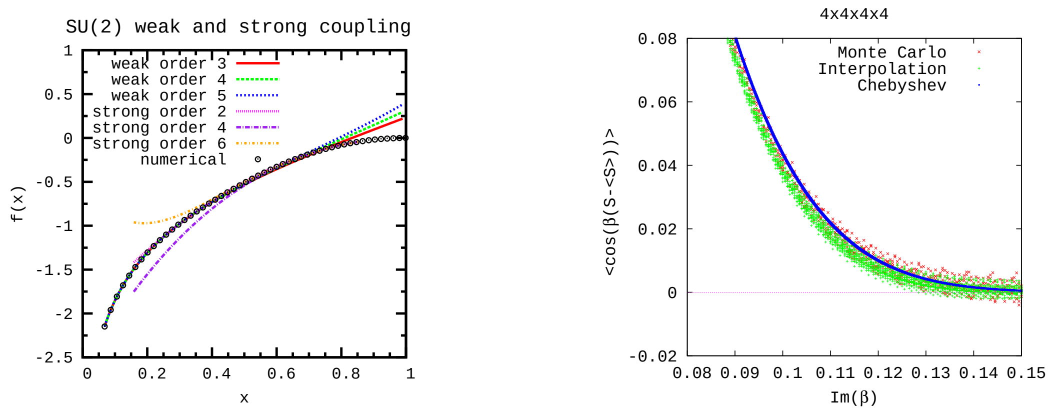

Figure 7: Weak and strong coupling expansion of f at a few intermediate orders (left) ; $< \cos(Im\beta(S- < S >)) >$ as a function of the imaginary part of β at fixed real part 2.18 with three methods: spline interpolation, Chebyshev fitting and Monte Carlo on a 4^4 lattice. The different sets are obtain by bootstraps.

$U(1)$ lattice gauge theory (A. Bazavov)

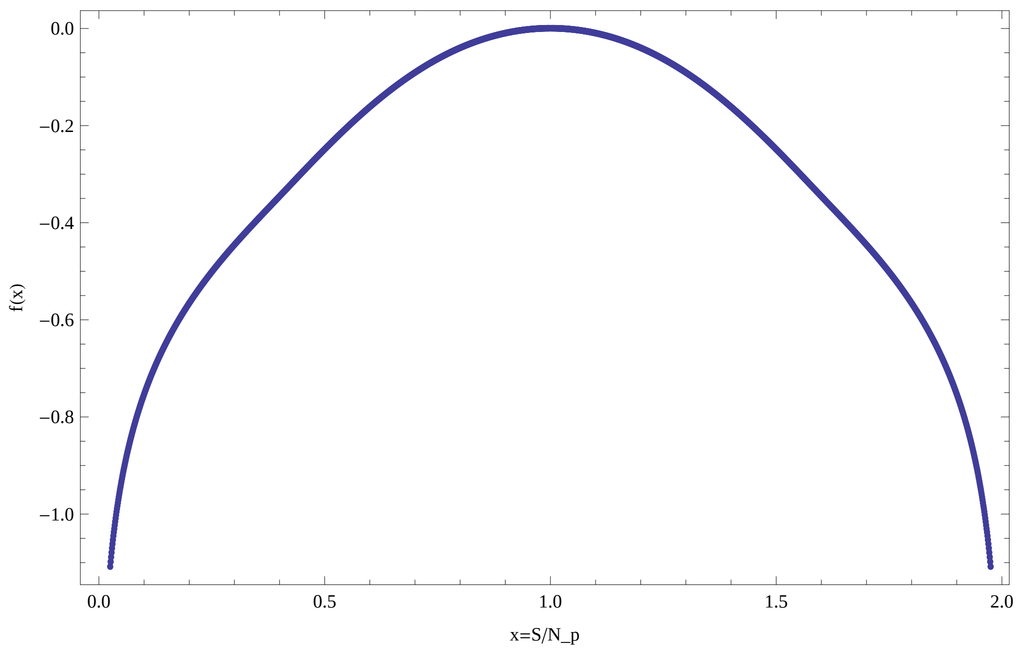

Figure 8: Density of states for $U(1)$ on a 4^4 lattice by multicanonical methods.

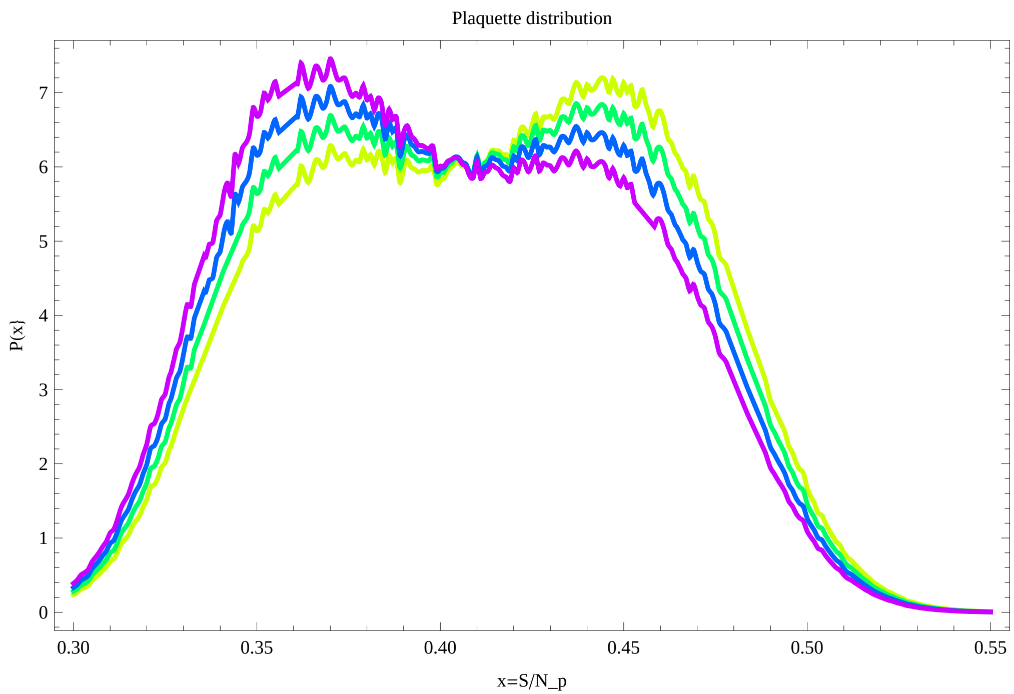

Figure 9: Plaquette distribution for $U(1)$ at β=0.978 (olive) , 0.979 (green), 0.98 (blue), and 0.981 (purple), using the density of states for a 4^4 lattice.

Finite Size Scaling for the deconfinement transition

In the study of the finite temperature phase transition of QCD, it is important to be able to localize precisely β_c and identify the universality class of the transition. This information can be obtained from the study of the so-called Binder cumulants of the Polyakov's loop denoted g_4. In recent studies, we noticed that localizing the intersection of curves at different volumes using linear fits can lead to inaccuracies. We studied g_4 on $N_\tau \times {N_\sigma}^3$ lattices for a pure SU(2) gauge theory. A. Denbleyker used a better β resolution than previous studies in intervals shrinking with the volume in order to reduce the nonlinear effects. We performed linear fits of $g_4 = a_L + b_L\beta$ around an approximate value of β_c that we call $(\beta_c)_{app}$ for different N_σ(L). We shrink the interval when N_σ increases: $|\beta - (\beta_c)_{app}| < \epsilon \times N_\sigma^{(-1/\nu)_{app}}$. ϵ is taken small enough to have negligible nonlinear corrections, typically 0.02. We can determine $1/\nu$ from the linear fit above. $b_L \simeq f_1 \times N_\sigma^{1/\nu}/\beta_c$. The fit have a slight dependence on $(\beta_c)_{app}$ and L. Liu constructed histograms for different ranges of values.

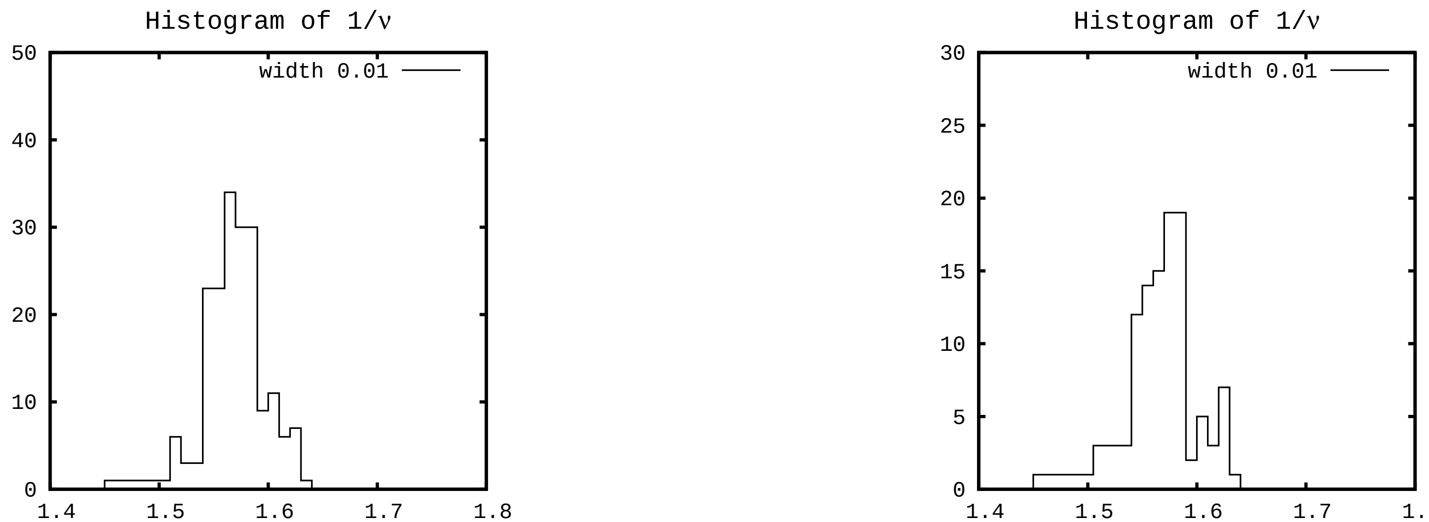

Figure 10: Left Fig.: $(\beta_c)_{app}$ changes from 2.297 to 2.301; $(1/\nu)_{app}$ changes from 1.4 to 1.8. $\overline{1/\nu} = 1.570$; $\sigma = 0.027$. Right Fig.: $(\beta_c)_{app}$ changes from 2.298 to 2.300; $(1/\nu)_{app}$ changes from 1.4 to 1.8. $\overline{1/\nu} = 1.571$; $\sigma = 0.028$.

Determination of the critical exponent ω Unless we determine β_c and $1/\nu$ very precisely, it is very difficult to subtract the effects of the third term of Eq. (**??**). If we can work at β_c, this term is absent:

$$g_4(\beta_c, N_\sigma) = g_4(\beta_c, \infty) + c_0 N_\sigma^{-\omega} \tag{3}$$

Consistently with the previous section and the rest of the literature, we assume the universal value $g_4(\beta_c, \infty) = 0.46575$ as found in Ref. [**?**]. $Log[|g_4 - g_4(\beta_c, \infty)|]$ vs. $Log[N_\sigma]$ should be linear right at β_c and nonlinear for all the other βs. This is shown in Figure 11. At the same time, the slope is $-\omega$. The result we obtained from this analysis is $\omega = 2.030(36)$.

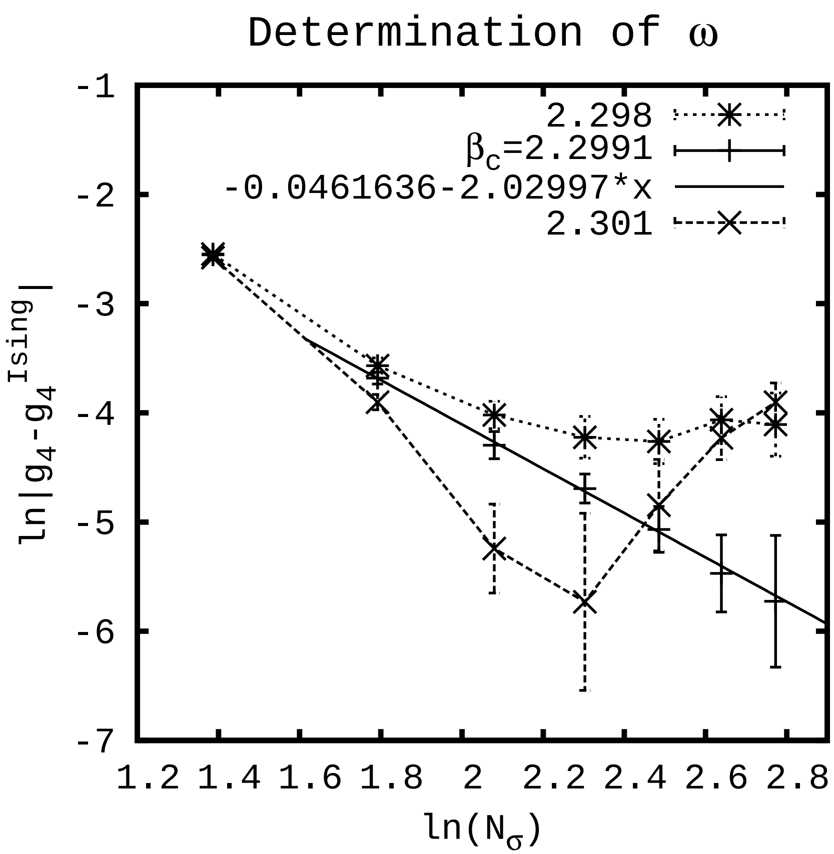

Figure 11: For β_c=2.2991, the behavior is approximately linear: $g_4 \simeq g_4(\beta_c, \infty) + c_0 \times N_\sigma^{-\omega}$.

This is very different from $\omega_{Ising} = 0.812$ [**?**]. It is possible that the coefficient of the $N_\sigma^{-\omega}$ is very small and the exponent we extrapolated is a sub-subleading exponent. For a detail discussion of the subleading corrections, see Ref. [**?**]. The most plausible explanation seems that this exponent is related to the irrelevant direction associated with the breaking of rotational symmetry [**?**] and which is close to 2.

Work on scalar models

We wrote a review article solicited by J. of Physics A, summarizing recent progress regarding nonlinear aspects of RG flows for scalar models in the hierarchical approximation (which can be seen as a local potential approximation). The review stresses the necessity to find the connection with methods where the renormalization group transformation evolves continuously. In the later case, the improvement of the local potential approximation is well developed and could be compared with the improvement method of the hierarchical approximation also outlined in the review article. We have recently designed methods which can interpolate between the two approaches (with Y. Liu, see below).

We have calculated the critical exponents for a variable number of blocked sites ℓ^D (calculations were done for $D = 3$). The results are numerically stable when the number of sites blocked is integer (see Fig. 12 which shows a simple power law.). We are working on an interpolation method to obtain results in the non-integer case and take the limit of infinitesimally close to 1.

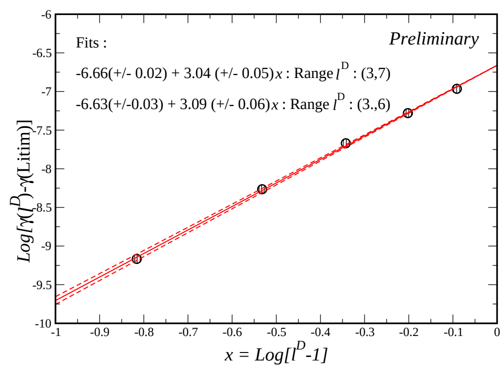

Figure 12: $\ln(\gamma(\ell^D) - \gamma(1))$ versus $\ln(\ell^D - 1)$.

Finite size scaling for scalar models

In 2.2.d, we stressed the complications due to nonlinear effects in the study of Binder cumulants. We studied this problem for the hierarchical Ising model where the statistical errors are negligible. The order of magnitude of the nonlinear effects can be estimated from data at relatively small volume. Using this estimate, we proposed to use linear fits in increasingly small temperature regions as the volume is increased (rather than using a fixed temperature interval). The choice of the exact coefficient of proportionality ϵ (see 2.2.d) can be optimized and reveals remarkable crossing patterns among estimates illustrated in Fig. 13.

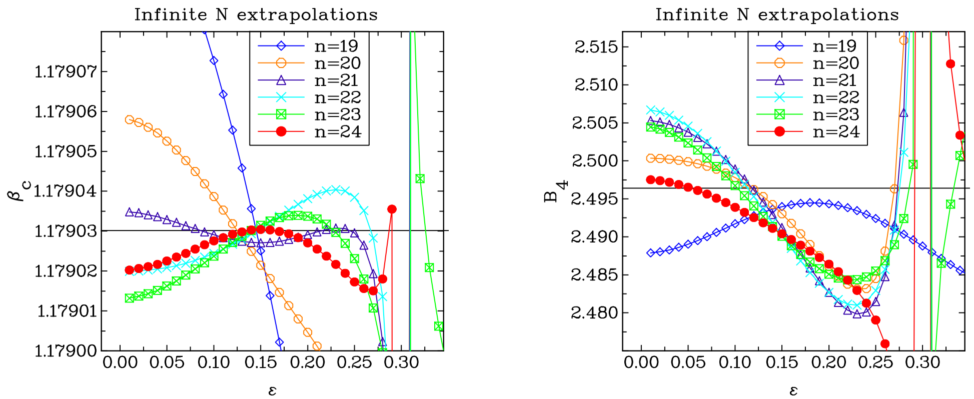

Figure 13: Infinite volume extrapolations of β_c and B_4 as a function of the optimization parameter ϵ. The horizontal line are accurate numerical values.

Dyson instability for $2D$ nonlinear $O(N)$ sigma models

For lattice models with compact field integration (nonlinear sigma models over compact manifolds and gauge theories with compact groups) and satisfying some discrete symmetry, the change of sign of the bare coupling g_0^2 at zero results in a mere discontinuity in the average energy rather than the catastrophic instability occurring in theories with integration over arbitrarily large fields. This indicates that the large order of perturbative series and the non-perturbative contributions for these models should have unexpected features. Using the large-N limit of 2-dimensional nonlinear $O(N)$ sigma model, we studied the complex singularities of the average energy for complex 't Hooft coupling $\lambda^t = g_0^2 N$. A striking difference with the usual situation is the absence of cut along the negative real axis. The zeros of the partition function can only be inside a clover shape region of the complex λ^t plane, or outside a region of the $1/\lambda^t$ plane bounded by 4 approximate hyperboloids and outlined in Fig. . We calculated the density of states at infinite volume and in the saddle point approximation and used the result to verify numerically the statement about the zeros. We proposed dispersive representations of the derivatives of the average energy for an approximate expression of the discontinuity. The discontinuity is purely non-perturbative and contributions at small negative coupling in one dispersive representation are essential to guarantee that the derivatives become exponentially small when $\lambda^t \to 0^+$.

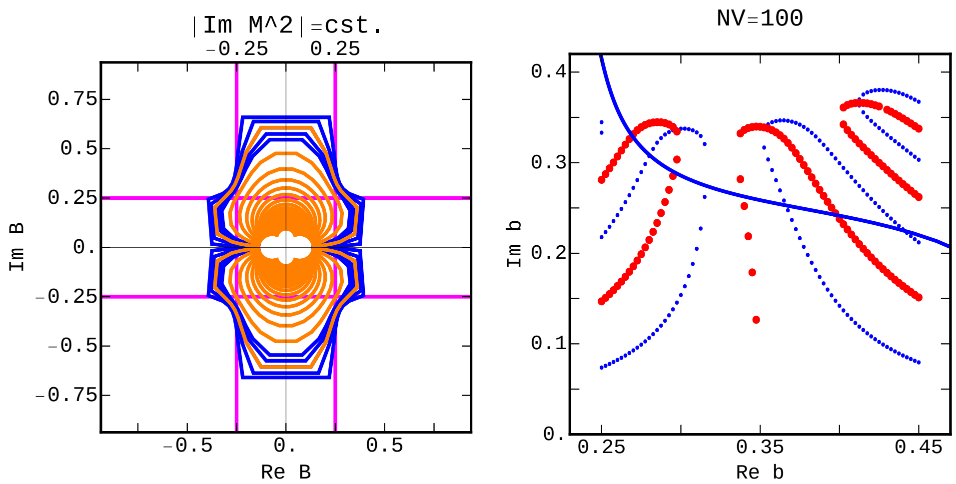

Figure 14: Complex values taken by $B(M^2) = 1/\lambda^t(M^2)$ calculated in the saddle point approximation when M^2 varies over the complex plane (here on horizontal lines in the M^2 plane with spacing 0.1 (black, blue) and 0.5 (gray, orange)). Fisher's zeros for $NV = 100$. Zeros of ReZ (small dots, blue), zeros of ImZ (larger dots). The solid line (blue) is the image of a horizontal line slightly below the cut in the M^2 plane.

In the argument for the absence of Fisher's zero inside the region bounded by the approximate hyperboloids visible in Fig. and in the numerical determination of the zero above, we have used polynomial approximations of the density of states at infinite volume. We would like to confirm these results by taking the limit of large volume of numerical calculations done at finite volume. This would also help understanding the scaling observed in the gauge case. This work is being done with H. Zou.

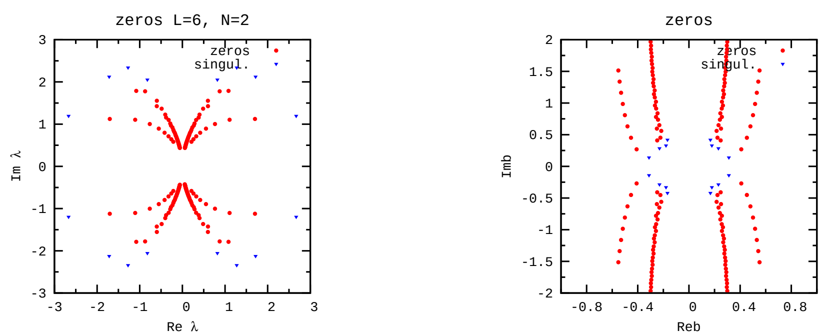

Figure 15:

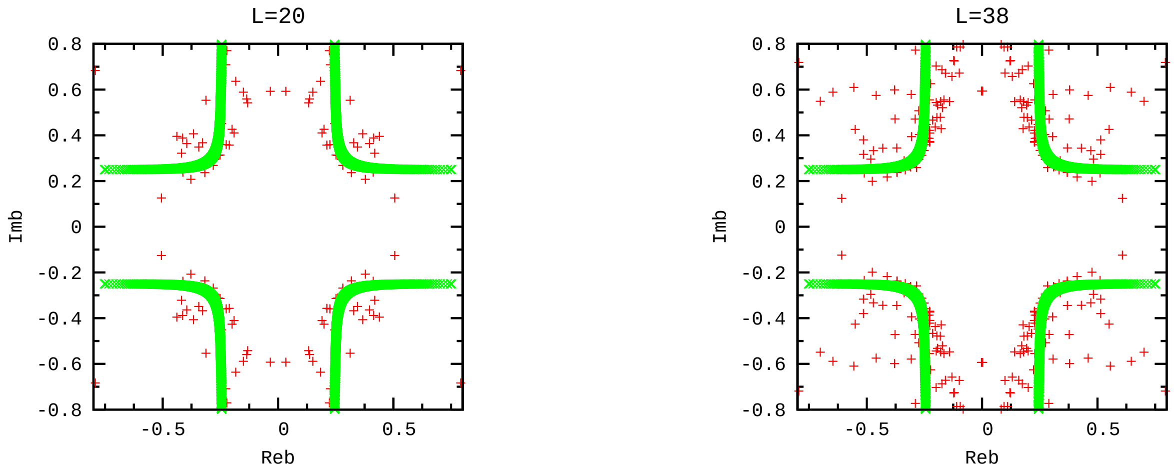

Figure 16: