Department of Energy Progress report 2004-2006

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General Motivations

- Large distance effects in QCD (crucial for masses, decay constants and mixing angles) can only be handled with Monte Carlo simulations, but getting rid of lattice effects and finite size effects is costly (the "Berlin-Wall"):

$$\text{Cost} \propto (\text{Lattice size})^5 \, (\text{Lattice spacing})^{-7}$$

- The short distance effects in QCD can be described by perturbation theory (asymptotic freedom), however QCD series diverge fast. For the hadronic width of the Z^0, the term of order α_s^3 is more than 60 percent of the term of order α_s^2 and contributes to one part in 1,000 to the total width (a typical experimental error at LEP).

- Next Leading 0rder (NLO) and NNLO corrections can be significant for processes relevant for the LHC.

  For instance, $pp \to Z$+4 jets (Zvi Bern talk at QCD 2006)

  For NNLO corrections in LHC processes, see a recent talk of K. Ellis: http://theory.fnal.gov/people/ellis/Talks/wab.pdf

  Nobody knows if the next order will improve or worsen the accuracy!

  (And such calculations can take years!)

- The experimental error bars of the anomalous magnetic moment of the the muon a_μ have been shrinking. We are reaching the limit of perturbative accuracy in theoretical estimates. Precision tests may become a major source of information regarding new laws of nature in the long term.

Quantity	*1990 Value* ($\times 10^{11}$)	*2001 Value* ($\times 10^{11}$)	*Change* ($\times 10^{11}$)
$a_\mu^{\rm QED}$	116 584 695.5(5.4)	116 584 705.7(2.9)	$+10.2$
$a_\mu^{\rm Had}$ (vac. pol 1)	7 068(59)(164)	6 924(62)	-144
$a_\mu^{\rm Had}$ (vac. pol 2)	-90(5)	-100(6)	-10
$a_\mu^{\rm Had}$ (light by light)	49(5)	-85(25)	-134
$a_\mu^{\rm EW}$ (1 loop)	195(10)	195(10)	0
$a_\mu^{\rm EW}$ (2 loop)	—	-43(4)	-43
$a_\mu^{\rm SM}$ (total)	116 591 918(176)	116 591 597(67)	-321

Table 1: Improvements in the theoretical calculation of a_μ from 1990 to 2001. The major shifts were primarily due to errors in the earlier calculations, new calculations of higher order effects, improved $e^+e^- \rightarrow$ hadrons and tau data, and additional utilization of perturbative QCD. (From W. Marciano, hep-ph/0105056)

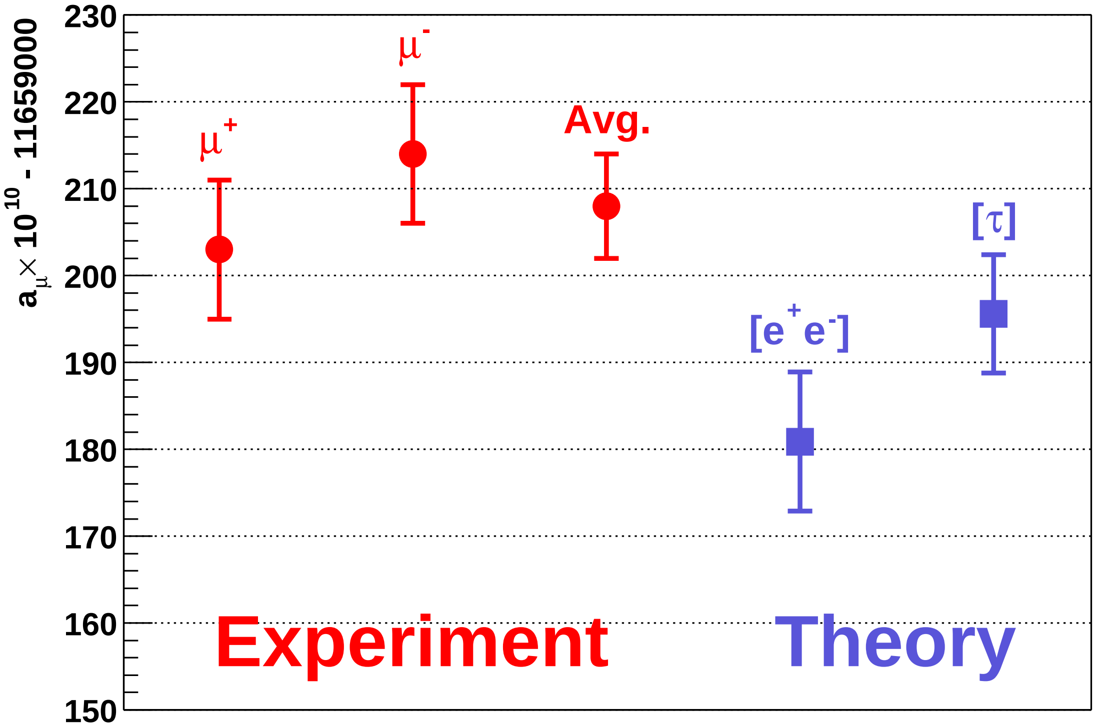
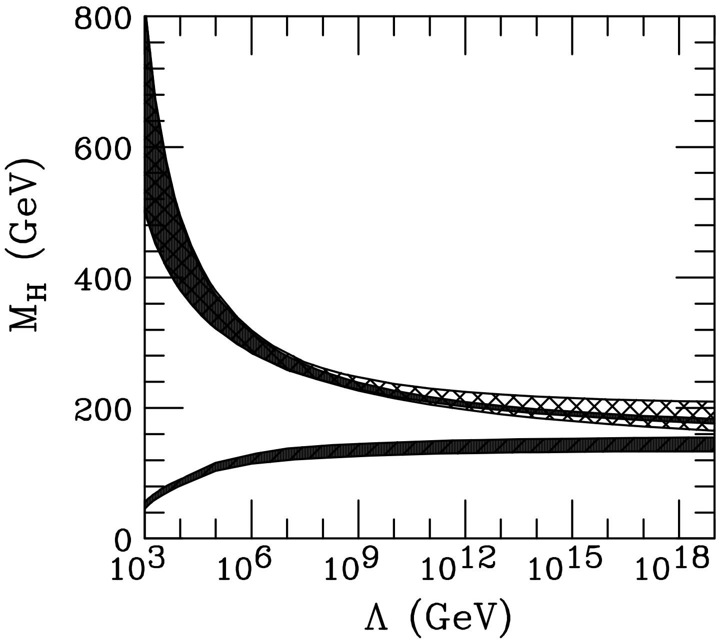


Figure 1: Measurements of a_μ by E821 with the SM predictions (Bennett et al., Phys. Rev. Lett. 92 161802 (2004); hep-ex/0401008).

- Perturbation theory imposes stability and triviality bounds on the Higgs mass m_H. If we believe these perturbative bounds, m_H should be in a small window near $m_H = 175\ GeV$, unless there is new physics at a scale Λ (see Figure below from T. Hambye and K. Riesselmann, Phys. Rev. D 55, 7255, 1997). However, these bounds probably reflect the failure of perturbation theory (K. Holland, Lattice 2004)



Summary

- There is an urgent need to develop new analytical methods based on weak coupling perturbative expansions, but modified in such way that they can be extended beyond the range of conventional methods.
- The main problem of perturbation theory is that it cannot succesfully handle large quantum fluctuations (the large field contributions in the path integral).
- The large field configurations become important in the crossover region between weak and strong coupling (the region of interest for QCD!).
- The renormalization group method is essential to make calculations in the crossover region.

Main Scope of our Effort

- We develop new field theoretical methods applicable in situations where conventional perturbative methods (Feynman diagrams) fail.

- We mostly work in the framework of the lattice formulation of scalar and gauge theories (where numerical tests are possible).

- We improve existing expansions (weak coupling, strong coupling) by controlling the large field configurations.

- We use the renormalization group method (which relates the behavior on small lattices to the behavior on larger lattices) whenever possible.

Big Goals

- "User friendly" modified Feynman rules for the standard model.

- Ab initio calculations justifying the phenomenological successes of Operator Product Expansion based approaches (relying on the gluon condensate).

- Generic numerical methods based on the renormalization group and applicable for scalar models (calculation of exponents, tests of triviality/stability bounds).

Recent Accomplishments

- Understanding of the large order behavior of the perturbative expansion of the average plaquette (the lattice analog of the gluon condensate) in quenched QCD. The apparent singularities are off the real axis (no third-order phase transition). Mean field theory explains the series up to order 20. Infrared renormalons probably dominate beyond order 25.

- Semi-classical parametrizations of the non-perturbative part of the plaquette and the β-function in quenched QCD.

- Universal behavior of the modified perturbative coefficients (calculated numerically with a large field cutoff) in quantum mechanics.

- Numerical treatment of the crossover between two fixed points of the renormalization group transformation of the hierarchical model (accurate calculation of non-universal renormalization group invariants).

- Approximate equivalence between the renormalization group transformation of the hierarchical model and Polchinski's equation in the local potential approximation (exponents differ by less than 10^{-5}).

- Numerical calculation of critical effective potentials and understanding of their complex singularities.

Recent Publications

1. Y. MEURICE, The non-perturbative part of the plaquette in pure gauge theory, Phys.Rev. **D 74**, 096005, 2006.

2. L. LI, and Y. MEURICE, About a possible 3rd order phase transition at T=0 in 4-D gluodynamics, Phys.Rev. **D 73**, 036006, 2006.

3. J. J. GODINA, L. LI, and Y. MEURICE and M. B. OKTAY, High-accuracy critical exponents of O(N) hierarchical sigma models, Phys.Rev. **D 73**, 047701, 2006.

4. L. LI, and Y. MEURICE, Asymptotically universal crossover in perturbation theory with a field cutoff, J.Phys. **A 39**, 8681, 2006.

5. L. LI, and Y. MEURICE, A tractable example of perturbation theory with a field cutoff: The anharmonic oscillator, J. Phys. **A 38**, 8139, 2005.

6. L. LI, and Y. MEURICE, An Example of Optimal Field Cut in Lattice Gauge Perturbation Theory, Phys. Rev. **D 71**, 054509, 2005.

7. L. LI, and Y. MEURICE, Lattice Gluodynamics at Negative g^2, Phys.Rev. **D 71**, 016008, 2005.

8. Y. MEURICE, and M. B. OKTAY, Universality in Nontrivial Continuum Limits: a Model Calculation, Phys.Rev. **D 69**, 125016 , 2004.

9. Y. MEURICE, Small Numerators Canceling Small Denominators of The High Temperature Scaling Variables, Phys.Rev. **E 69**, 056108, 2004.

10. B. KESSLER, L. LI, and Y. MEURICE, New Optimization Methods for Converging Perturbative Series with a Field Cutoff, Phys.Rev. **D 69**, 045014 1-8, 2004.

Recent Talks:

1. A. DENBLEYKER, D. DU, Y. MEURICE (presenting author), and M. Naydes, Definition and parametrization of non-perturbative effects in quenched QCD, Lattice 2006, Tucson, Proc. of Sc. LAT2005:215, 2006.

2. Y. MEURICE, A RG analysis of the interpolation between weak and strong coupling, Talk given at the 3rd Conference on the Exact Renormalization Group, Lefkada, Greece, September 2006, http://www.cc.uoa.gr/ papost/Meurice.pdf

3. Y. MEURICE, At Which Order Should we Truncate Perturbative Series? Talk given at Continuous Advances in QCD 2006, Minneapolis, May 2006, hep-th/0608097.

4. Y. MEURICE, Interpolation between weak and strong coupling of the plaquette in quenched QCD, Talk given at the ECT, Trento, March 2006, http://www.ect.it/Meetings/ConfsWksAndCollMeetings/ConfWksDocument/2006/Talks/27-31March/meurice.pdf .

5. Y. MEURICE, Improving the accuracy of perturbative calculations by using a large field cutoff, Talk given at the Univeristé Catholique de Louvain, March 2006.

6. Y. MEURICE, Improving the accuracy of perturbative calculations by using a large field cutoff, Talk given at the Univeristé Libre de Bruxelles, March 2006.

7. Y. MEURICE, Large order behavior of series and global RG flows, talk given at Miami 2005, Miami, December 2005.

8. L. LI, and Y. MEURICE (presenting author), Is there a 3rd order phase transition in 4D gluodynamics?, Lattice 2005, Trinity College, Dublin, Proc. of Sc. LAT2005:258,2006 .

9. L. LI, and Y. MEURICE (presenting author), Effects Of Large Field Cutoffs In Scalar And Gauge Models, *Nucl. Phys. Proc. Suppl.* **140** (2005) 788–790.

10. Y. MEURICE, Effects of Large Field Cutoffs in Lattice Gauge Theory, talk given at Miami 2004, http://server.physics.miami.edu/ curtright/ Miami2004/Meurice.html.

11. Y. MEURICE, Role of Large Field Configurations in Perturbation Theory, Challenges of LFT, KITP, March 2005, http://online.itp.ucsb.edu/online/lattice05/meurice/ .

12. , Y. MEURICE, Effects of Large Field Cutoffs in Scalar and Gauge Theories, Talk given at Washington University, October 2004.

13. L. LI and Y. MEURICE (presenting author), Large Field Cutoffs in LGT, hep-lat/0411020; Workshop on QCD in Extreme Environments, Argonne Nat. Lab., June 2004.

14. L. LI and Y. MEURICE (presenting author), Limit Of Perturbative Coefficients Calculated With A Large Field Cutoff, Fermilab, June 2004, Nucl.Phys.Proc.Suppl.129:883-885, 2004.

Review Article (in progress)

A proposal for a review article "Global Aspects of the Renormalization Group Flows of Dyson's Hierarchical Model" has been accepted by Jour. of Phys. A. A detailed sketch of the article was forwarded to the Editorial board and received positive comments. 50 pages written up to now. The expected date of completion is December 2006 .

Undergraduate Student Involvement:

We insist on involving undergraduates in our research. Our supervised undergraduates go to top graduate schools. We also supervise summer students from other universities (REU organized by Prof. Reno).

- B. Kessler (B. S. 2004, now at UC Berkeley as a grad. student, Van Allen Research Award recipient)

- A. Lytle (B. S. 2004, now at UW Seattle as a grad. student, Van Allen Research Award recipient)

- J. Cook (B. S. 2005, REU student in 2004, now at UI Urbana-Champaign as a grad. student, Van Allen Research Award recipient)

- M. Snyder (B. S. 2006 from Elon University, REU student in 2005, now at NC State as a grad. student)

- A. Denbleyker (B. S. 2006, now in our department as a grad. student, Van Allen Research Award recipient)

- M. Naides (B. S. expected in 2008 from Cornell U at Ithaca, REU student in 2006)

Graduate Students Involvement:

We support graduate students with a mixture of teaching assistantships and research assistantships.

Recent Ph. D.:

- B. Oktay (Ph. D. 2001; postdoc at University of Illinois at Urbana Champaign; now postdoc at Trinity College, Dublin; current collaborator).

- L. Li (Ph. D. 2005; recipient of the Goertz-Nicholson award in May 2001; employed by Apache Design in Mountain View, California)

Current students

- Daping Du (came in 2005, passed the qualifier exam, TA/RA)
- Alan Denbleyker (new graduate student in 2006, TA)
- Liu Yuzhi (new graduate student in 2006, TA)

The University has strict requirements regarding the oral competency of graduate students employed as Teaching Assistant beyond the first year. In the past three years, three of our graduate students were unable to pass the test before the end of the first year.

We would like to send some students to the Lattice Summer School in Seattle in August 2007 or TASI 2007.

Computational Facilities

In 2003, we built a 16 node cluster that has allowed us to create large numbers of gauge configurations on lattices with up to 16^4 sites and to work on scalar field theory in various dimensions. Due to several recent hardware failures, this cluster will be phased out.

We have built a new cluster with 8 single CPU nodes with 3.2 GHz Pentium4 processors and Gigabyte motherboards with a build-in fast ethernet card. It is now being reconfigured with Oscar 5.0 (released this month) on openSUSE 10.0.

Multiprocessors computers have been build commercially and are low maintenance items. We would like to try 4 CPU units to use for distributed calculations. This type of workstation costs approximately 5,000 dollars.

Basic ideas (scalar case)

$$\int_{-\infty}^{+\infty} d\phi e^{-\frac{1}{2}\phi^2-\lambda\phi^4} \neq \sum_{0}^{\infty} \frac{(-\lambda)^l}{l!} \int_{-\infty}^{+\infty} d\phi e^{-\frac{1}{2}\phi^2} \phi^{4l} \tag{1}$$

The peak of the integrand of the r.h.s. moves too fast when the order increases. On the other hand, if we introduce a field cutoff, the peak moves outside of the integration range and

$$\int_{-\phi_{max}}^{+\phi_{max}} d\phi e^{-\frac{1}{2}\phi^2-\lambda\phi^4} = \sum_{0}^{\infty} \frac{(-\lambda)^l}{l!} \int_{-\phi_{max}}^{+\phi_{max}} d\phi e^{-\frac{1}{2}\phi^2} \phi^{4l} \tag{2}$$

General expectations: for a finite lattice, the partition function Z calculated with a field cutoff is convergent and $\ln(Z)$ has a finite radius of convergence.

One plaquette LGT

$$Z(\beta, N) = \int \prod_{l \in p} dU_l \mathrm{e}^{-\beta(1 - \frac{1}{N} ReTrU_p)} \ ,$$

$$Z(\beta, 2) = (2/\beta)^{3/2} \frac{1}{\pi} \int_0^{2\beta} dt t^{1/2} \mathrm{e}^{-t} \sqrt{1 - (t/2\beta)}$$

modified partition function:

$$Z(\beta, 2, t_{max}) = (2/\beta)^{3/2} \frac{1}{\pi} \int_0^{t_{max}} dt t^{1/2} \mathrm{e}^{-t} \sqrt{1 - (t/2\beta)}$$

$$Z(\beta, 2, t_{max}) = (\beta\pi)^{-3/2} 2^{1/2} \sum_{l=0}^{\infty} A_l(t_{max})(2\beta)^{-l} \ ,$$

with

$$A_l(t_{max}) \equiv \frac{\Gamma(l+1/2)}{l!(1/2-l)} \int_0^{t_{max}} dt e^{-t} t^{l+1/2} \ ,$$

When $t_{max} \rightarrow \infty$ the integral becomes the (complete) Γ function and the coefficients grow factorially. In lattice perturbation theory, we "add the tails".

Note: $t_{max} = 2\beta$ means β-dependent coefficients.

When t_{max} is finite, the integral is bounded by a power of t_{max}. When $t_{max} \leq 2\beta$, the sum converges.

The need for interpolation

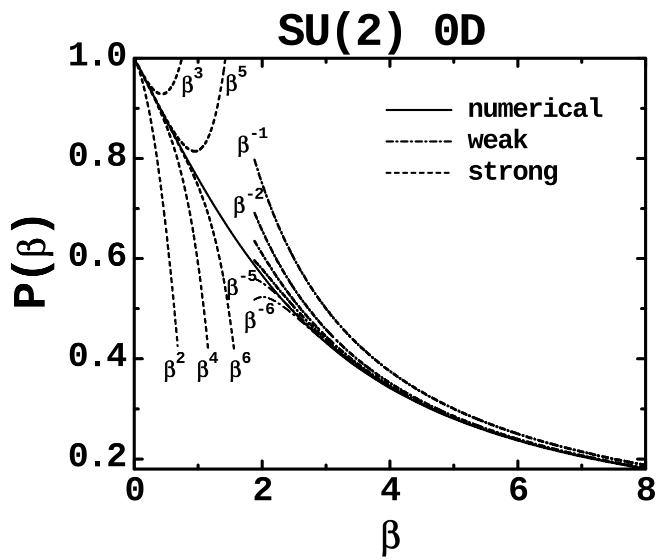


Figure 2: P versus β for $SU(2)$ on one plaquette. The solid line represents the numerical values; the dashed lines on the left, successive orders in the strong coupling expansion; the dot-dash lines on the right, successive order in the weak coupling expansion.

Optimal t_{max}

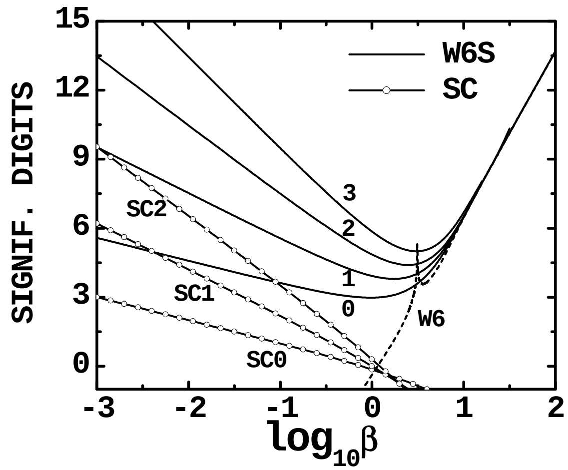


Figure 3: Significant digits obtained from the weak series truncated at order 6, calculating t_{max}/β using the strong coupling expansion at order 0 to 3, compared to the weak coupling expansion at order 6 (dotted line W6) and the strong coupling expansion at order 0 to 2 (empty circles SC). Hopefully it can be extended to $4D$ where calculations are much harder!

$4D$ Quenched QCD

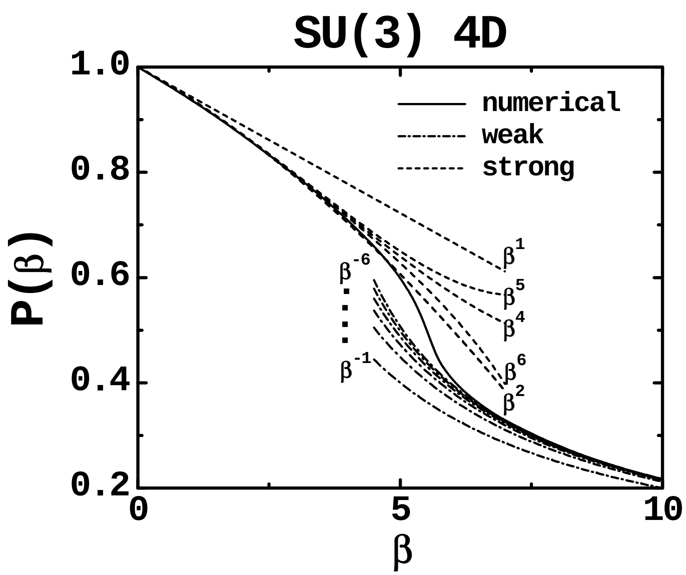


Figure 4: P versus β for $SU(3)$ in 4 dimensions. The solid line represents the numerical values; the dashed lines on the left, successive orders in the strong coupling expansion; the dot-dash lines on the right, successive orders in the weak coupling expansion. $P \sim < F^a_{\mu\nu} F^{a\mu\nu} >$, the non-perturbative part is often called gluon condensate. ($\beta = 6$ corresponds to 0.1 fermi).

Discontinuity at $g^2 \to \pm 0$

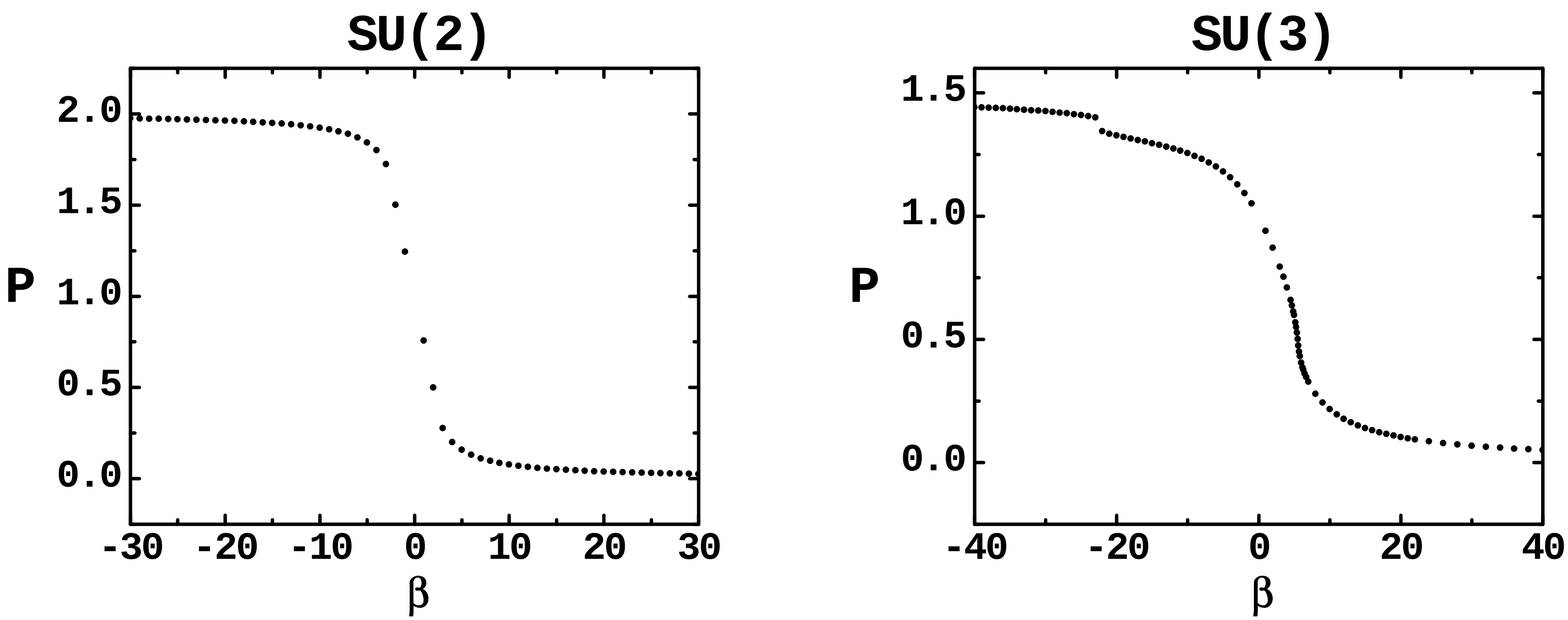
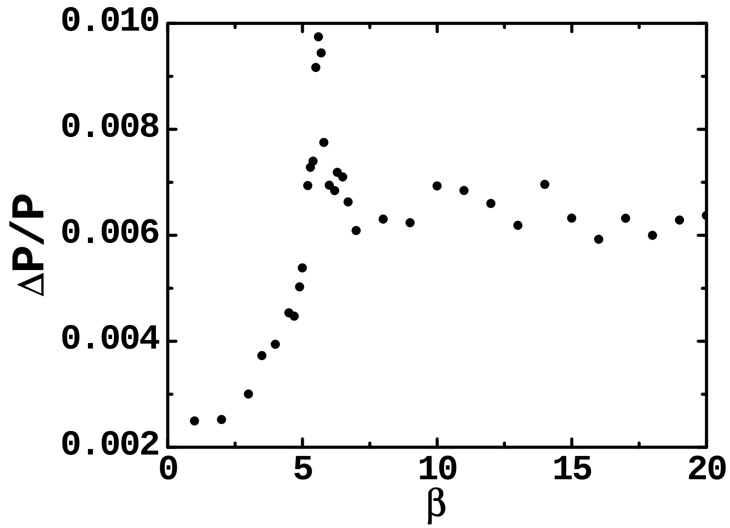


Figure 5: MC calculation of the average action density $P(\beta)$ for $SU(2)$ and $SU(3)$. Does the gap control the large order behavior? (Daping Du)

Effect of a gauge invariant cut on P

The effect of the cut is very small but of a different size below, near or above $\beta = 5.6$. The relative change of the configuration average of P when 80 percent of the large field configurations are discarded, for various values of β in a pure $SU(3)$ LGT on a 8^4 lattice is shown below (this illustrates the sensitivity to large field configurations in the crossover region).



Singularities in the complex coupling plane

Series analysis suggest a nonanalytical power behavior near $\beta = 6/g^2 \simeq 5.8$ with a specific heat exponent $\alpha \simeq -0.1$ and an (unexpected) singularity in the second derivative of P, or in other words in the third derivative of the free energy (third order phase transition).

The peak in the third derivative of the free energy present on 4^4 lattices disappears if the size of the lattice is increased isotropically up to a 10^4 lattice . On the other hand, on $4 \times L^3$ lattices, a jump in the third derivative persists when L increases. Its location coincides with the onset of a non-zero average for the Polyakov loop and the known location of the finite-temperature transition.

The absence of evidence for a peak with height increasing with the volume on isotropic lattices and the absence of massless states for Wilson's action in the fundamental representation, can be resolved by moving the singularity in the complex $1/\beta$ plane. If the imaginary part of the location of the singularity Γ is within the range $0.001 < \Gamma < 0.01$, it is possible to limit the second derivative of P within an acceptable range without affecting drastically the behavior of the perturbative coefficients.

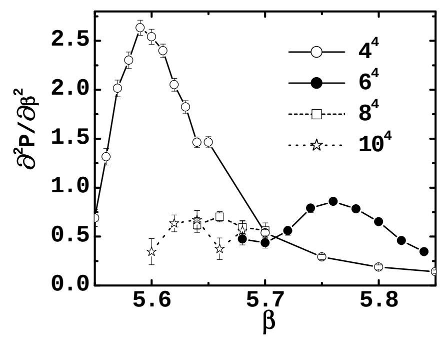


Figure 6: Second derivative of P versus β for $4^4,\ 6^4,\ 8^4$ and 10^4 lattices.

Searches for zeroes of the partition function

Alan Denbleyker has searched for the zeroes of the partition function in the complex β plane by using the reweighting method. The bounds in the $1/\beta$ plane discussed above imply that the the location of the zero closest to the positive real axis in the β plane should have an imaginary part between 0.03 and 0.3 when the volume becomes large enough. If the imaginary part is larger than $K\sqrt{\ln(N_{conf.})/V}$ with $N_{conf.}$ the number of configurations and K a calculable function of the real part of order 1 varying slowly, the determination of the zeroes becomes inaccurate. For small volume (4^4), we were able to establish the existence of a zero clearly within the radii of confidence of various reweighting near $\beta = 5.55 \pm i0.1$ in agreement with results on $4L^3$ lattices. For larger (6^4 and 8^4) lattices, we have not found zeroes clearly within the radii of confidence. Up to now, everything seems consistent with the bound $|Im\beta| > 0.03$.

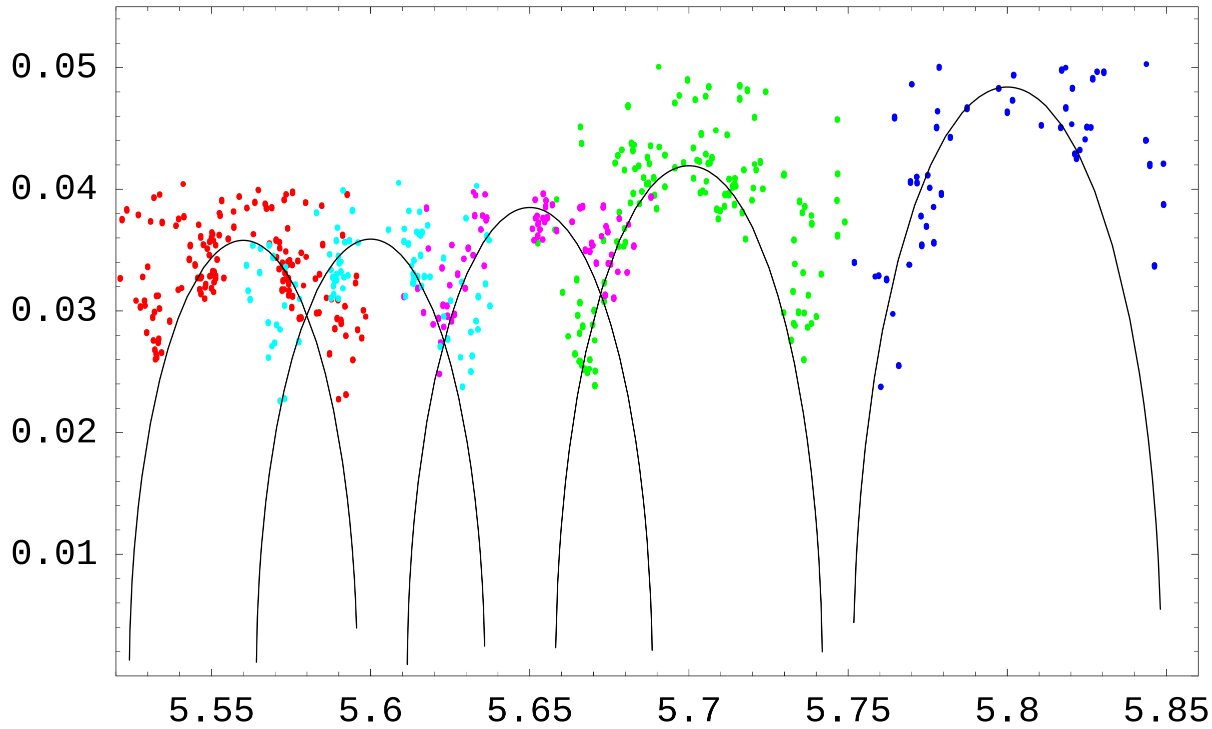


Figure 7: Zeroes of the partition function in the complex β plane for a 8^4 lattice. The dots correspond to distinct bootstraps and the solid lines to the radii of confidence.

Effect of adjoint term in the action (in collaboration with A. Velytsky (UCLA)).

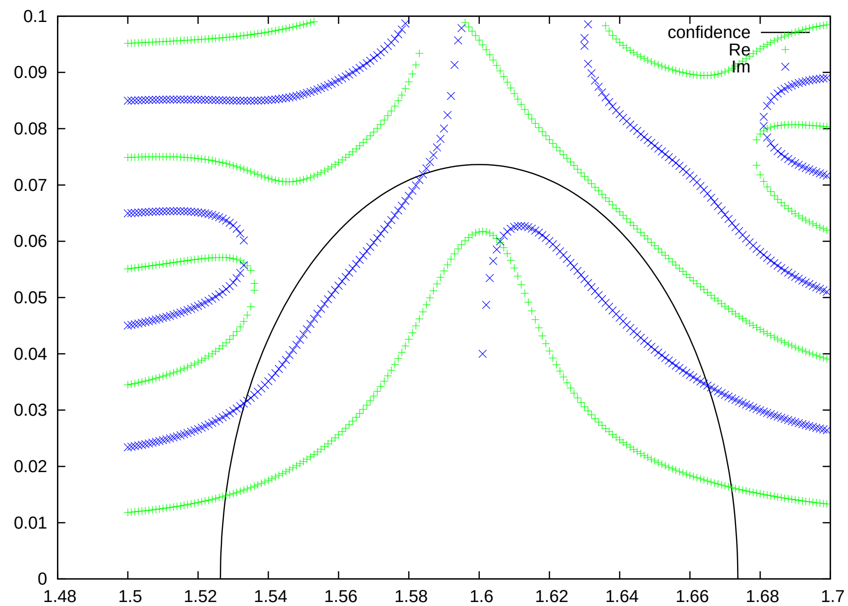


Figure 8: Zeroes of the partition function for $SU(2)$ on a 4^4 lattice in the β_f plane and $\beta_a = 0.7$

Definition and parametrization of the non-perturbative part of the plaquette

Non-perturbative effects are often invoked in phenomenological applications. Successful examples can be found in SVZ sum rule calculations. In this approach, the gluon condensate plays an important role. In lattice gauge theory, numerical non-perturbative calculations are possible, however the notion of gluon condensate is sometimes a source of controversy in part because the separation between perturbative and non-perturbative part is ambiguous. We have studied a particular type of separation with various examples. We defined the non-perturbative part of a quantity as the difference between its numerical value and the perturbative series truncated by dropping the order of minimal contribution and the higher orders. For the anharmonic oscillator, the double-well potential and the single plaquette gauge theory, the non-perturbative part can be parametrized as $A \, \lambda^B \, \mathrm{e}^{-C/\lambda}$

and the coefficients A, B and C can be calculated analytically. For lattice QCD in the quenched approximation, the perturbative series for the average plaquette is dominated at low order by a complex singularity in the complex coupling plane and the asymptotic behavior can only be reached by using extrapolations. The first extrapolation is based on the mean field assumption that the pseudo specific heat has a logarithmic singularity in the complex β plane. Integrating, we obtain

$$\sum_{k=1} a_k \beta^{-k} \simeq C(\mathrm{Li}_2(\beta^{-1}/(\beta_m^{-1} + i\Gamma)) + \mathrm{h.c}\ , \tag{3}$$

For the intermediate allowed value $\Gamma = 0.003$, $C = 0.0654$ and β_m=5.787 yield exact values for coefficients 9 and 10. Except for the first term, the agreement with the other coefficients is very good. The fact that such a good agreement can be reached by tuning two parameters begs for a diagrammatic explanation! This extrapolation (and another one based on

IR renormalons dominance) that provide a consistent description of the series up to order 20-25, favor the idea that the non-perturbative part is a power of the force scale (S. Necco and R. Sommer, Nucl. Phys. B662, 328, 2002). We proposed a parametrization of the force scale as the two loop universal terms with exponential corrections.

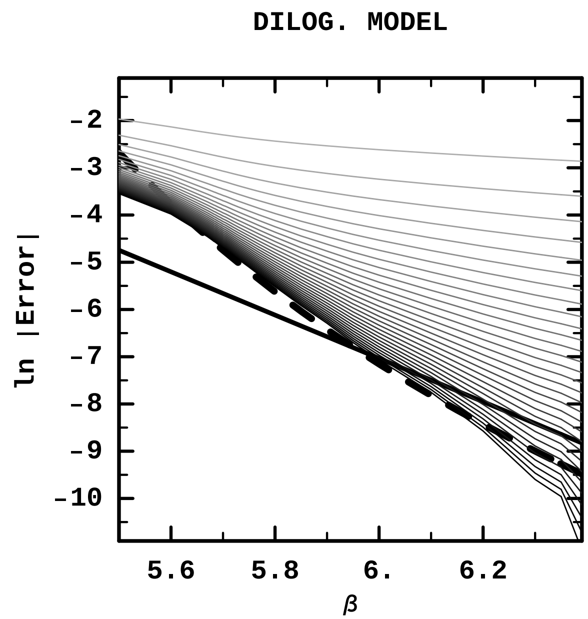


Figure 9: Natural logarithm of the absolute value of the difference between the series and the numerical value for order 1 to 30 for quenched QCD with the dilogarithm model. As the order increases, the curves get darker. The long dash curve is ln$(0.65\ (a/r_0)^4)$. The solid curve is ln$(3.1 \times 10^8 \times (\beta)^{204/121-1/2} \mathrm{e}^{-(16\pi^2/33)\beta})$

Work on Scalar Models

The limitations of perturbation theory are well understood for scalar field theories. Large field configurations have little effect on commonly used observables but are important for the average of large powers of the field and dominate the large order behavior of perturbative series. A simple way to remove the large field configurations consists in restricting the range of integration for the scalar fields in the path integral. The method produces convergent series in nontrivial cases.

The simplest quantum mechanical example where this method can be applied is the anharmonic oscillator. Recently, we have treated this example in complete detail. Quantum mechanics can be seen as quantum field theory with one time dimension and zero space dimension. However, we will use the usual x notation (instead of ϕ) in the following.

The anharmonic oscillator with a “field” cut x_{max}

$$H = \frac{p^2}{2} + V(x) \ ,$$

with

$$V(x) = \begin{cases} \frac{1}{2}\omega^2 x^2 + \lambda x^4 & \text{if} \quad |x| < x_{\max} \\ \infty & \text{if} \quad |x| \geq x_{\max} \end{cases}$$

$$E_0(x_{max}) = \omega \sum_{k=0}^{\infty} E_0^{(k)}(x_{max})(\lambda/\omega^3)^k \ ,$$

$$R_k(x_{max}) \equiv E_0^{(k)}(x_{max})/E_0^{(k)}(\infty) \ ,$$

finite radius of convergence: $\lambda_c \simeq 65 x_{max}^{-6}$

Universal shape of the modified coefficients as a function of the field cutoff

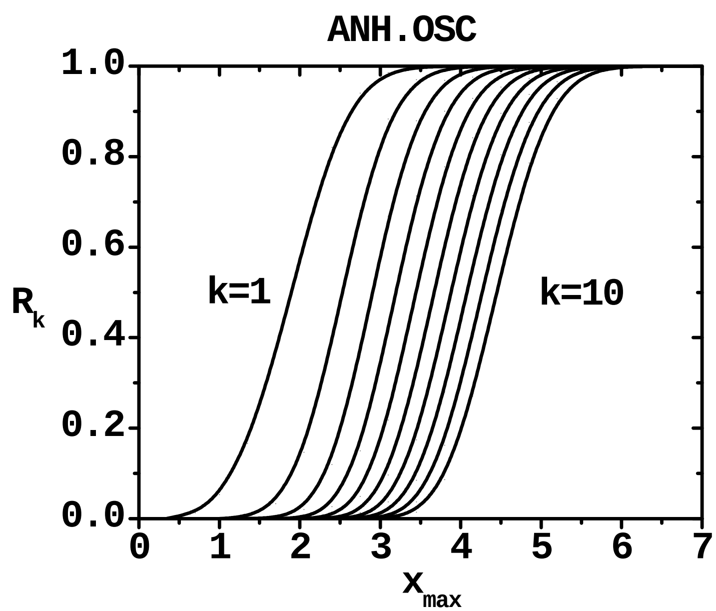


Figure 10: $R_k(x_{\rm max}) = E_0^{(k)}(x_{max})/E_0^{(k)}(\infty)$ for k going from 1 to 10.

Asymptotic data collapse

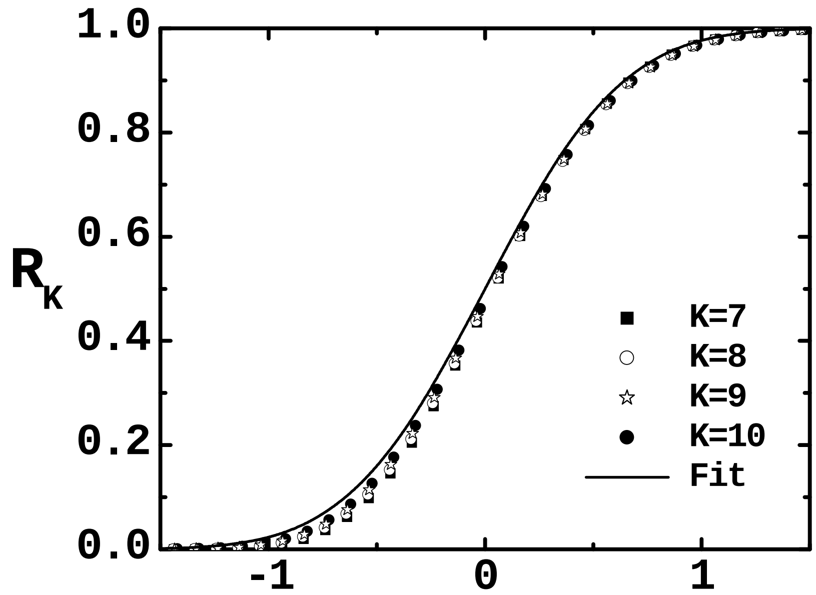


Figure 11: $R_k(x_+ x_0(k))$ for $k = 7, \ldots 10$ and the function $U_{anh,1}(x)$.