Theoretical Physics Part II: Lattice Gauge Theory

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with Don Sinclair (ANL/U.of Iowa),

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Content of the presentation

- Overview
- Scientific goals
- People
- Activities
- Facilities
- Recent progress
  1. Toward a Fisher's zero approach of the conformal window (Energy Frontier)
  2. B-physics beyond the standard model (Intensity Frontier)
  3. New methods in lattice field theory
- Models of a Strongly-Coupled Higgs Sector (by Don Sinclair)

Overview

- Main interest: Models of strong interactions primarily on the lattice
- Applications: QCD and extensions beyond the standard model
- Methods: MC simulations, improved perturbation theory and renormalization group techniques
- Senior Personnel: Don Sinclair (1/4 time at U. of Iowa, DOE funded)
- Graduate Students: 4 at U. of Iowa now (one of them at Fermilab with URA for AY 12-13), one recently graduated (now postdoc at U. Illinois Urbana Champaign)
- Computational facilities: Clusters here and at Fermilab, NERSC
- New computational possibilities explored: optical lattice realizations of lattice gauge theory (NOT DOE funded)

Success of the Standard Model (SM)

- Tree level gauge boson masses: $M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_W} = M_Z^2 \cos^2\theta_W$
- Low-energy data can be used to determine the input parameters ($\alpha,\ G_F$ and $\sin\theta_W$)
- 1982: W and Z discovered with consistent masses
- Precision measurements (LEP, ...) + radiative corrections imply that $m_t \approx 160 GeV/c^2$ (around 1990)
- Evidence for top quark (around 1994); 172 GeV/c^2 (2010)
- All the above + radiative corrections: $40 < M_H < 160 GeV/c^2$ (typical values found in PDG since 1999)
- 2011: Tevatron and LHC exclude most of the above region
- 2012: LHC discovery with $M_H \simeq 125 Gev/c^2$?
- No consistent and sustained hints for beyond the SM physics

2012: consistent hints for $M_H \simeq 125 Gev/c^2$ at LHC

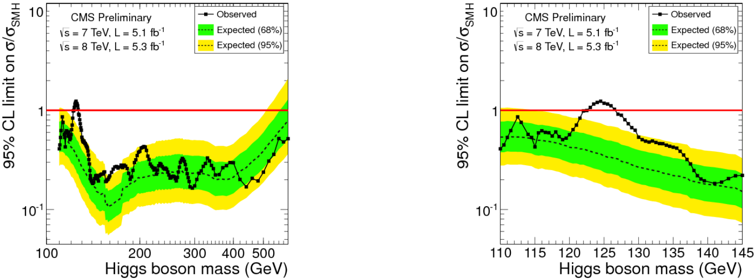


Figure: Most recent CMS σ/σ_{SMH} graphs (July 2012)

Important problems in particle physics

- Masses and mixing angles for neutrinos
- Accurate calculations of weak matrix elements; essential to observe hypothetical discrepancies with the SM
- Error estimates for applications of perturbative QCD
- Phase diagram of QCD at finite temperature and density
- Models for a light composite Higgs coming from hypothetical new gauge interactions at a multi-TeV scale; the difference in scale suggests an approximate conformal symmetry
- Why 3 generations?
- Gravity at short distance?

Note: around 1985, the HEP community put more emphasis on logical problems than on dynamical or computational problems. In 2012, dynamical and computational problems stand out.

The role of new theoretical methods in the next decade

Recent LHC results indicate that better methods to deal with strong interactions will become crucial to test the standard model or offer economical alternative to the standard Higgs mechanism. The only nonperturbative formulation known to define QCD or QCD-like theories is the lattice regularization.

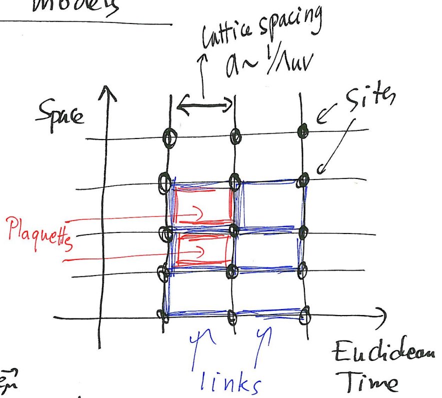
Monte Carlo simulations provide robust results for lattice gauge models. The field has been driven by fast progress in CPU and GPU, but the lattices remain small and the lattice spacing large, requiring difficult extrapolations. Ultimately, we need to find more analytical methods to understand the continuum limit and the infinite volume limit of these models especially in near conformal situations.

Lattice models

Matter fields

ψ_x : fermions
↳ site

ϕ_x : scalar
↳ site



Gauge fields

$$U_{\vec{x},\vec{x}+\vec{e}_\mu} \simeq e^{i\int_{\vec{x}}^{\vec{x}+\vec{e}_\mu} A_\mu ds}$$

link

Lattice Models Considered I: $O(N)$ nonlinear sigma models

The lattice sites are denoted **x** and the scalar fields $\vec{\phi}_{\mathbf{x}}$ are N-dimensional unit vectors. The partition function reads:

$$Z = C \int \prod_{\mathbf{x}} d^N \phi_{\mathbf{x}} \delta(\vec{\phi}_{\mathbf{x}}.\vec{\phi}_{\mathbf{x}} - 1) \mathrm{e}^{-\beta E[\{\phi\}]} \ ,$$

with

$$E[\{\phi\}] = -\sum_{\mathbf{x},\mathbf{e}} (\vec{\phi}_{\mathbf{x}}.\vec{\phi}_{\mathbf{x}+e} - 1) \ ,$$

with **e** running over the D positively oriented unit lattice vectors and $\beta \equiv (1/g_0^2)$

Lattice Models Considered II: $U(1)$ and $SU(N)$ Lattice Gauge Theory

The unitary matrices U_{link} are associated with the links (or bonds) of a cubic lattice.

$$Z = \prod_{links} \int dU_{link} \mathrm{e}^{-\beta \mathcal{S}} \ ,$$

with the Wilson action

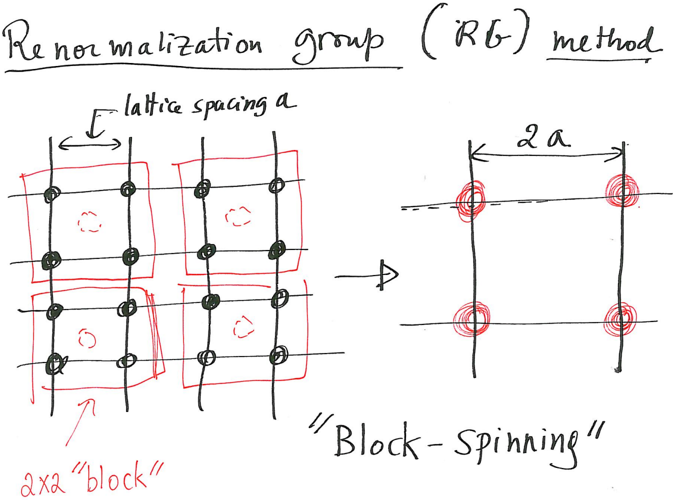
$$\mathcal{S} = \sum_{p} (1 - (1/N) ReTr(U_p)) \ .$$

and $\beta \equiv 2N/g^2$. U_p denotes the ordered products of 4 U_{link} along an elementary square (“plaquette"). We typically use periodic boundary conditions.

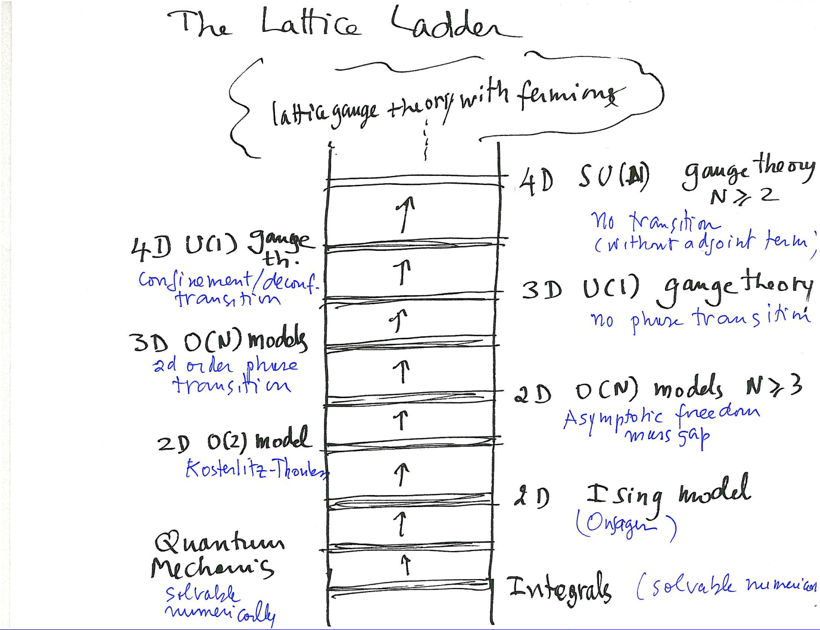
Methods

- Monte Carlo simulations (facilities are discussed below)
- We have made analytical progress in two directions:
  - Improving Feynman diagrams methods (control of the large field contributions in the path integral)
  - Improving Renormalization Group methods (removing the “walls" in blocking procedures)
- Developing new methods requires to go up on a “ladder" of lattice models (Integrals $\rightarrow$ Quantum mechanics $\rightarrow$ 2D Ising model $\rightarrow$ .....$\rightarrow$ 4D Gauge theories with fermions (see below)).

The Renormalization Group(RG) method



The lattice theory ladder



The conformal window

- The possibility of having a strongly coupled and composite Higgs sector has motivated searches for nontrivial infrared fixed points in asymptotically free gauge theories.
- The location of the conformal windows, the region in parameter space where a nontrivial infra-red fixed point exists, for several families of models has been the subject of many recent investigations. When approached on the low N_f side this is a strongly interacting problem that can only be treated with Lattice Gauge Theory.
- A situation of particular phenomenological interest is when the β function for a new hypothetical gauge coupling approaches zero from below and starts “walking".

Complex RG flows

- The near conformal situation of a walking coupling constant can be obtained in some examples by varying a parameter in such a way that two RG fixed points coalesce and disappear in the complex plane.
- This motivated us to study extensions of the Renormalization Group (RG) flows in the complex coupling plane.
- In all examples considered, we found that the Fisher's zeros act as "gates" for the RG flows ending at the strongly coupled fixed point.
- The Fisher's zeros are the zeros of the partition function in the complex inverse coupling or inverse temperature plane (later, we use the "β-plane" terminology).
- This is a complex extension of the general picture of confinement proposed by Tomboulis.

Senior Personnel: Don Sinclair

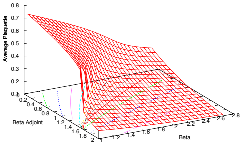
- Pioneer in finite temperature QCD, composite Higgs models and numerical methods to calculate fermion determinants
- Special Time Appointment (< 50 percent) at Argonne
- 25 percent appointment at U. of Iowa, funded by the DOE sinceJuly 2012
- Recent work: $SU(3)$ with 2 and 3 sextets (next talk)
- Shares codes and computational facilities with us
- Played an essential in our recent calculations of Fisher zeros for $SU(3)$ with 4 and 12 quarks flavors

Daping Du (former graduate student)

Daping Du came in fall 2005 and earned his Ph. D. degree in June 2011. Attended the Seattle Lattice Gauge Theory summer school in 2007. He worked with the Fermilab Lattice group with a URA fellowship from January to August 2011 and calculated the fragmentation fractions for the B meson. He is now a postdoc at the University of Illinois in Urbana. He worked on the fits of plaquette distribution, saddle point estimates of the Fisher zeros and interpolations for the density of states in $U(1)$ and $SU(2)$ gauge theories. He developed new algorithms for histogram reweighting and search for zeros.

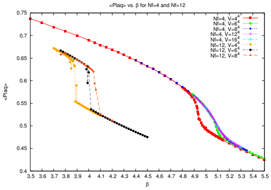
Alan Denbleyker (graduate student)

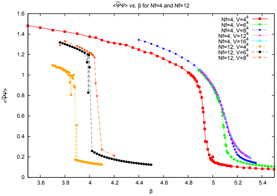
Alan Denbleyker came in fall 2006. Attended the Seattle Lattice Gauge Theory summer school in 2007. He works on MC simulations in $SU(2)$ gauge theories with and without adjoint terms and works on histogram reweighting and finite size scaling to compare finite temperature and bulk transitions. He is the system manager for our cluster and repository. Has been supported as a T.A. during the academic year and as a R.A. during summer. He has passed the qualifying exam and will take the comprehensive exam soon. This year he is R. A. in summer and fall. Graduation expected in May 2013.



Yuzhi “Louis" Liu (graduate student)

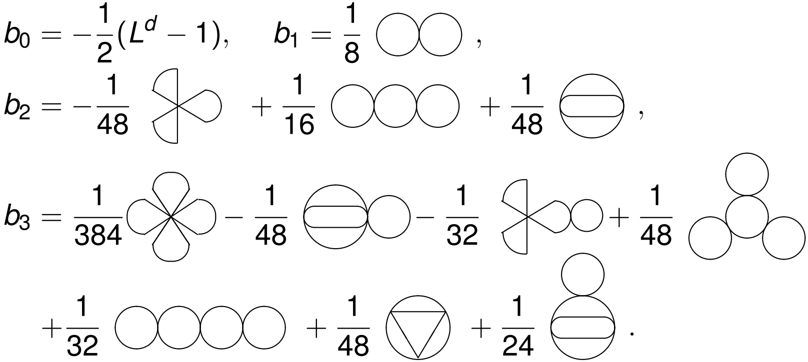
Yuzhi “Louis" Liu came in fall 2006. He has passed the qualifying and comprehensive exams and the T.A. certification. Attended the Les Houches Lattice Gauge Theory summer school in 2009 and participated in many workshops (KITPC, INT, Fermilab). He worked on the comparison between discrete renormalization group methods that we have been using and continuous limits of these methods used by other authors. He was partially supported by the University as a R. A. to work on optical lattice calculations. He is working on multiflavor gauge theories and on $B_s \to K\mu\nu$. Now at Fermilab in AY with a URA award. Graduation expected in June 2013.





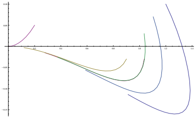
Haiyuan Zou (graduate student)

Haiyuan Zou came in fall 2008. He has passed the qualifying exam and the T. A. certification. He has been working on improved perturbation theory and complex renormalization group flows in nonlinear sigma models. He has learned conventional perturbative methods for W-production with Prof. Reno. He has been supported partially as a T.A. and as a R.A.



Judah Unmuth-Yockey (graduate student)

Judah Unmuth-Yockey came in fall 2011 and joined the group in summer 2012. He attended the Seattle summer school on lattice gauge theory. He has been running the multicanonical code for $U(1)$ gauge theory. He works on the Renormalization group flows in the Migdal-Kadanoff approximation and its improvement using Tensor Network methods. He has been supported partially as a T.A. and partially as a R.A.



Publications in Journals

- Y. Meurice, Approximate recursions for for tensor renormalization, preprint in preparation.
- Jon A. Bailey, A. Bazavov, C. Bernard, C.M. Bouchard, C. DeTar, Daping Du, A.X. El-Khadra, J. Foley, E.D. Freeland, E. Gamiz , Steven Gottlieb, U.M. Heller, Jongjeong Kim (AS. , A.S. Kronfeld, J. Laiho, L. Levkova, P.B. Mackenzie, Y. Meurice, E.T. Neil, M.B. Oktay, Si-Wei Qiu, J.N. Simone, R. Sugar, D. Toussaint, R.S. Van de Water, and Ran Zhou, *Refining new-physics searches in $B \to D\tau\nu$ decay with lattice QCD*, Phys. Rev. Lett. **109** , 071802 (2012) (Editor's Suggestion). http://prl.aps.org/abstract/PRL/v109/i7/e071802
- Y. Meurice, Remarks about Dyson's instability in the large-N limit, e-Print: arXiv: 1203.2256 [hep-th]. http://arXiv.org/pdf/1203.2256.pdf

- Jon A. Bailey, A. Bazavov, C. Bernard, C.M. Bouchard, C. DeTar, Daping Du, A.X. El-Khadra, J. Foley, E.D. Freeland, E. Gamiz , Steven Gottlieb, U.M. Heller, Jongjeong Kim (AS. , A.S. Kronfeld, J. Laiho, L. Levkova, P.B. Mackenzie, Y. Meurice, E.T. Neil, M.B. Oktay, Si-Wei Qiu, J.N. Simone, R. Sugar, D. Toussaint, R.S. Van de Water, and Ran Zhou, *$B_s \to D_s/B \to D$ semileptonic form factor ratio and their application to $BR(B_s^0 \to \mu^+\mu^-)$*, Phys. Rev. **D 85**, 114502 (2012). http://prd.aps.org/abstract/PRD/v85/i11/e114502
- A. Bazavov, B. Berg, Daping Du, and Y. Meurice, *Density of States and Fisher's zeros in $U(1)$ pure gauge theory*, preprint arXiv:1202.2109, Phys. Rev. **D 85** 056010 (2012).
- Y. Liu and Y. Meurice, *Lines of Fisher's zeros as separatrices for complex renormalization group flows*, Phys. Rev. **D 83** 096008 (2011).
- Y. Meurice and H. Zou, *Complex RG flows for 2D nonlinear O(N) sigma models*, Phys. Rev. **D 83** 056009 (2011).

- Y. Meurice, R. Perry, and S.-W. Tsai, Editors of the theme issue: *New applications of the renormalization group method in physics*, Phil. Trans. R. Soc. **A 369** (2011).
- Y. Meurice, R. Perry, and S.-W. Tsai, *New applications of the renormalization group method in physics, a brief introduction*, Phil. Trans. R. Soc. **A 369** 2602 (2011).
- A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, and Haiyuan Zou, *Fisher's zeros as boundary of renormalization group flows in complex coupling spaces*, Phys. Rev. Lett. **104** 251601 (2010).

Recent talks

- Toward dynamical gauge fields on optical lattices, Y. Meurice (Program Talk) given at “Critical behavior of lattice models in atomic and molecular, condensed matter and particle physics", KITPC, July 2012.
- Fisher zeros and conformality in lattice models, Yannick Meurice, with A. Bazavov, B. Berg, A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, D. Sinclair, J. Unmuth-Yockey, Haiyuan Zou, Lattice 2012.
- Local gauge symmetry on optical lattices? Yuzhi Liu, Yannick Meurice and Shan-Wen Tsai, Poster at Lattice 2012.
- Y. Meurice, “Renormalization group approach of scalar field theory", U. of Rochester, April 2012.

Recent talks

- Y. Meurice, “Fisher’s zeros, complex RG flows and confinement in lattice models", Miami 2011, Dec. 2011.
- Y. Meurice, “Fisher’s zeros, complex RG flows and confinement in LGT models" , (APS-Prairie, Nov. 2011).
- Yannick Meurice, “QCD calculations with optical lattices?", Lattice 2011, (July 2011).
- Y. Meurice, “Confinement, RG flows in the complex coupling plane and Fisher’s zeros", CAQCD (Minneapolis May 2011),
- Y. Meurice, “Confinement and Walking Coupling Constants: A Renormalization Group Point of View", Argonne Nat. Lab. (May 2011).
- Y. Meurice, "Fisher’s zeros as the Boundary of RG flows in complex coupling space", UCLA, October 15, 2010.
- Y. Meurice, "Fisher’s zeros as the Boundary of RG flows in complex coupling space", UC Riverside, October 18, 2010.

- Y. Meurice, "Fisher's zeros as boundary of RG flows in complex coupling space", 5th ERG Conference, Corfu, September 14, 2010.
- Y. Meurice, "Dynamical Gauge Fields on Optical Lattices : A Lattice Gauge Theorist Point of View", (poster) KITP Conference: Frontiers of Ultracold Atoms and Molecules, Oct 11-15, 2010 .
- Y. Meurice, "Fisher's zeros as boundary of RG flows in complex coupling space", Univ. of Utrecht , August 10, 2010.
- Y. Meurice, "Fisher's zeros as boundary of RG flows in complex coupling space", Lattice 2010, Villasimius, June 18 2010.
- Y. Meurice, "Complex RG flows", Aspen Center for Physics, June 9, 2010.
- Y. Meurice, "Renormalization Group in the Complex Domain", Washington University, St Louis, March 17, 2010.
- Y. Meurice, "Complex zeros of the beta function, confinement and discrete scaling", New applications of the renormalization group method in nuclear, particle and condensed matter physics, Institute for Nuclear Theory Seattle. February 22 - 26, 2010.

Conference organization

- *New Applications of the Renormalization Group Method*, INT workshop, Feb. 22-26, 2010, with M. Birse, and S.-W.Tsai ; 35 participants, including 2 U. Iowa students.
- *Critical Behavior of Lattice Models*, Aspen Workshop, May 24 -June 11 2010, with G. Baym, U. Schollwoeck and S.-W. Tsai; 43 participants.
- *Critical behavior of lattice models* Kavli Institute for Theoretical Physics in China in July 24-August 31 2012. The International Coordinating Board is Lu-ming Duan (U. Michigan), Yannick Meurice (U. Iowa), Shan-Wen Tsai (UC Riverside), Xiao-gang Wen (MIT) and Zhenghan Wang (MicrosoftQ).
- Aspen Center for Physics, Summer 2013, “Lattice Gauge Theory in the LHC Era" (coming up)

Computer facilities

- 2003: first 16 node cluster.
- 2006: new cluster with 8 single CPU nodes having 3.2 GHz Pentium 4 processors and Gigabyte motherboards with a build-in fast ethernet card .
- 2010: new cluster with 10 nodes with 4GB of Ram, 2.33Ghz Core2 Quad processors, sata hard drives. The combined cost was $3337 or $334 per node. Built by A. Denbleyker.
- 3 Proposals of level C at Fermilab
- NERSC (0.5M core-hours in 2012)
- Helium: a large cluster at the University with 3508 computing cores across 359 nodes built in 2011 using pooled grant money from various groups at the University, and is maintained by the Universities ITS department. We have currently able to use it, and have requested 1M hours.

Our cluster

Recent progress

1. Toward a Fisher's zero approach of the conformal window
   - $O(N)$ nonlinear sigma models
   - $U(1)$ gauge theory
   - $SU(2)$ gauge theory
   - $SU(3)$ with fermions
2. B-physics beyond the standard model
   - $B_s \to \mu^+\mu^-$
   - $B \to D\tau\nu$
   - $\bar{B}_s \to K^+\mu^-\bar{\nu}$
3. New methods in lattice field theory focused on 3D and 4D $U(1)$
   - improved perturbative methods
   - RG methods
   - New: Tensor Network methods

Toward a Fisher's zero approach of the conformal window

- Fisher's zeros
- $O(N)$ nonlinear sigma models
- $U(1)$ gauge theory
- $SU(2)$ gauge theory
- $SU(3)$ with fermions

Fisher's zeros, Finite Size Scaling and the Conformal Window

Decomposition of the partition function (Niemeijer and van Leeuwen)

$$Z = Z_{sing.} e^{G_{bounded}}$$
$$Z_{sing.} = e^{-L^D f_{sing.}}$$

RG transformation: the lattice spacing a increases by a scale factor b

$$a \rightarrow ba$$
$$L \rightarrow L/b$$
$$f_{sing.} \rightarrow b^D f_{sing.}$$
$$Z_{sing.} \rightarrow Z_{sing.}$$

Important Conclusion (Itzykson et al. 83)

The zeros of the partition functions are RG invariant

Fisher's zeros: zeros of the partition function in the complex β plane

$Z_{sing.}$ in terms of scaling variables

We consider discrete RG transformations

Example

$b = 2$, for a sigma model on D-dimensional cubic lattice: 2^D fields are replaced by one blocked field

Lattice size (in a units)

$$L \to L/b$$

Scaling variables (e. g. $u = \beta - \beta_c + \dots$, note: $\beta \propto 1/g^2$)

$$u_i \to \lambda_i u_i$$

Relevant variables: $\lambda_i = b^{1/\nu_i}$; Irrelevant variables: $\lambda_j = b^{-\omega_j}$

RG invariance of $Z_{sing.}$

$$Z_{sing.} = Q(\{u_i L^{1/\nu_i}\}, \{u_j L^{-\omega_j}\})$$

Zeros for one relevant variable

For a single relevant variable $u \simeq \beta - \beta_c$, we have $Z_{sing} = Q(uL^{1/\nu})$.

The complex equation $Z = 0$ can be written as two real equations for two real variables and generic solutions are isolated points.

$$Z = 0 \Rightarrow uL^{1/\nu} = w_r \text{ with } r = 1, 2, \ldots$$

This implies the approximate form for the zeros:

$$\beta_r(L) \simeq \beta_c + w_r L^{-1/\nu}$$

There are many examples, where these discrete solutions follow approximate lines or lay inside cusps. In the infinite volume limit, the set of zeros may (or may not) separate the complex plane into two or more regions.

For a first order transition: $\nu \to 1/D$.

“Confining" flows: 2D $O(N)$ models in the large-N limit

Complex extension of Tomboulis picture of confinement: the RG flows go directly from weak coupling to strong coupling (mass gap).

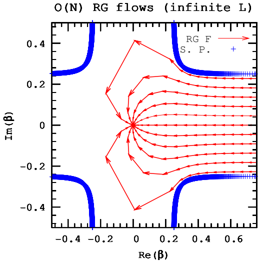


Figure: Infinite volume RG flows (arrows). The blending blue crosses are the β images of two lines of points located very close above and below the $[-8, 0]$ cut of the large-N “running" $\beta(M^2)$ in the M^2 plane. Fisher's zeros stay outside of the blue lines (YM, PRD 80 054020).

3D $U(1)$: no zeros near the real axis (Denbleyker)

3D $U(1)$ is confining. There is a gap in the spectrum and the zeros. See X-G. Wen's book p. 265 for a discussion of confinement and duality with the XY model.

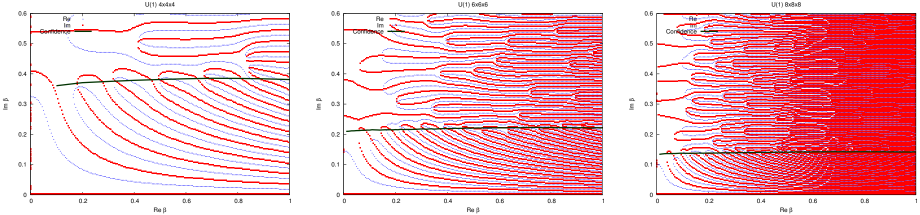


Figure: Fisher's zeros for $U(1)$ on L^3 lattices (L=4, 6 and 8 from left to right) The zeros of the real (imaginary) part are represented by the blue (red) curves and the region of confidence is below the green line (zeros near or above this line are not reliable).

4D $U(1)$: first or second order?

$U(1)$ on L^4: the average plaquette distribution has a double peak distribution with equal heights at a pseudo-critical β_S. For small L, the distance between the peaks slowly decreases with the volume. (PRD 85 with Bazavov, Berg and Daping Du).

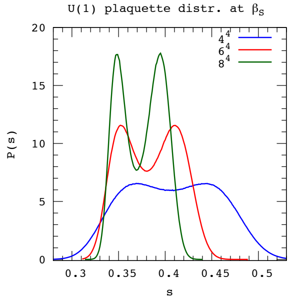
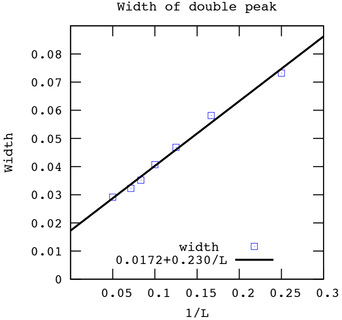
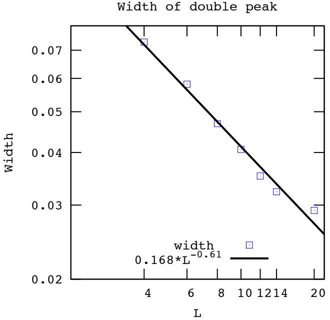


Figure: Average plaquette distribution for $U(1)$ at β_S for L= 4, 6 and 8.

4D $U(1)$: first or second order?

In the infinite volume limit, the width of the double peak distribution of the average plaquette goes to a nonzero limit (latent heat) for a first order phase transition and to zero as an inverse power of L for a second order transition. Better statistics for the large volumes are necessary to discriminate between the two scenarios.





4D $U(1)$ (with Bazavov, Berg and Du)

The complex zeros appear at the intersections of ReZ=0 and ImZ=0. Results obtained by integrating a reweighted density of states calculated with multicanonical methods (arxiv 1202.2109, PRD 85)

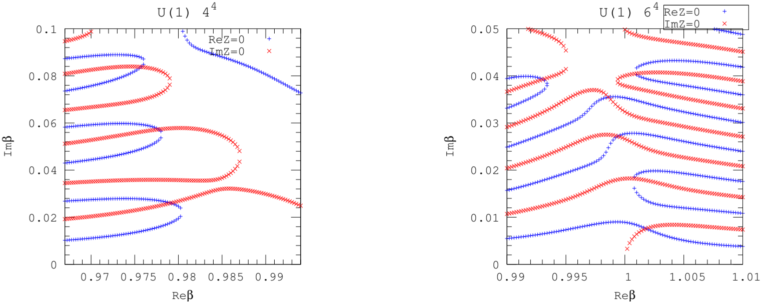


Figure: Zeros of the Re (+, blue) and Im (x, red) part of Z for $U(1)$ using the density of states for 4^4 and 6^4 lattices. Fits favor 1st order, larger volumes are needed.

$SU(2)$ with $\beta_{Adjoint}$ (with A. Denbleyker and Daping Du)

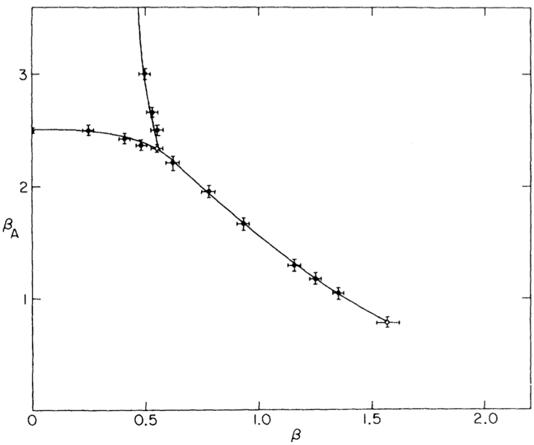
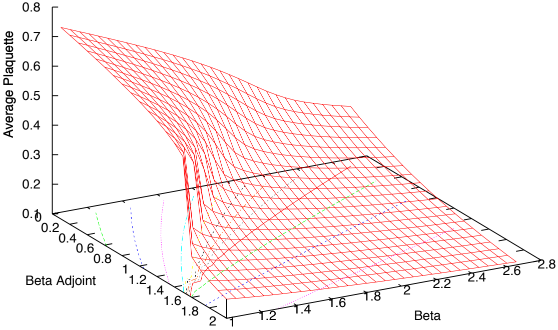


Figure: The Creutz-Bhanot phase diagram (Phys. Rev. D 24, 3212-3217).

$SU(2)$ with $\beta_{Adjoint}$ (with A. Denbleyker and Daping Du)



$SU(2)$ with $\beta_{Adjoint}$ (with A. Denbleyker and Daping Du)

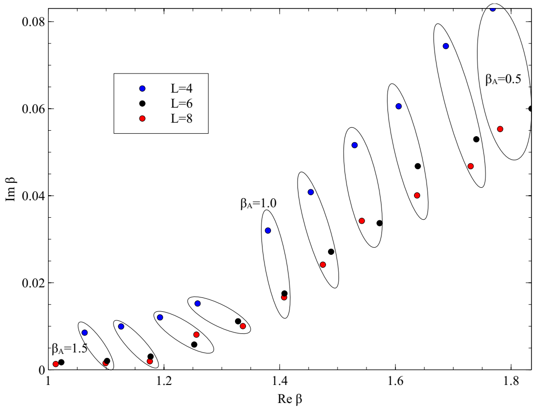
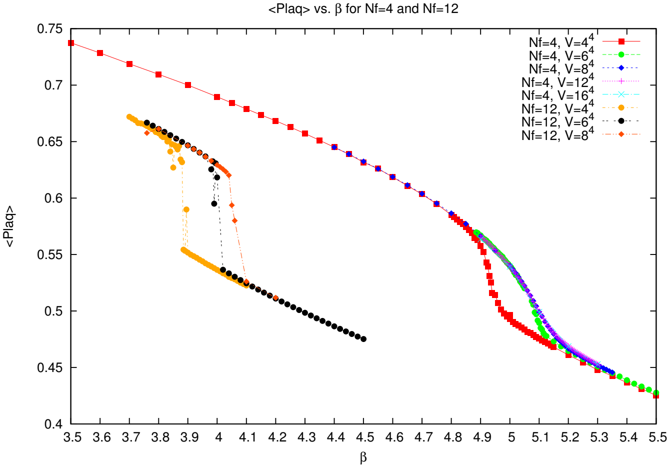


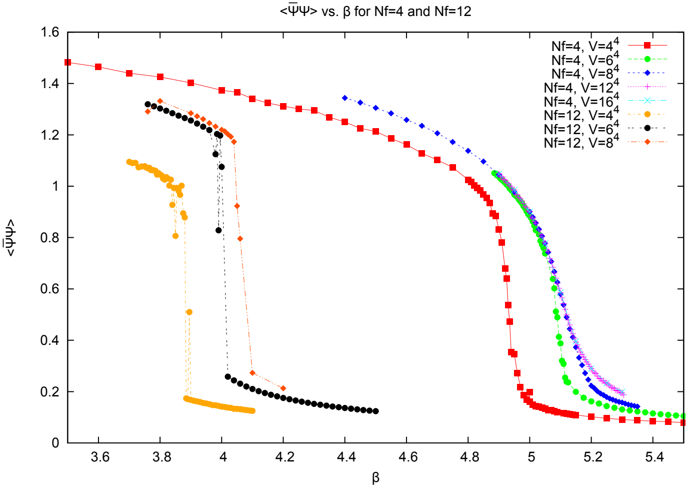
Figure: Lowest zeros for $\beta_{Adjoint}$= 0.5, 0.6, ..., 1.5. The robustness of these results are discussed in Daping's Du thesis.

$SU(3)$ with N_f= 4 and 12 (with Yuzhi Liu and Don Sinclair)



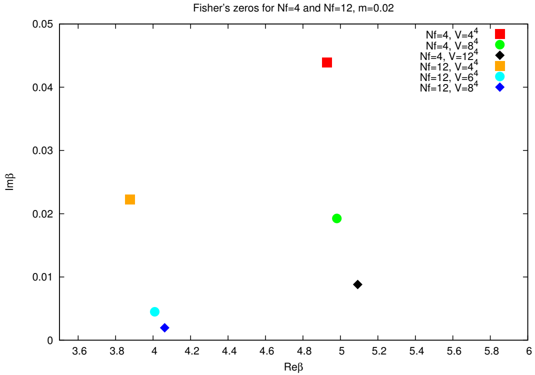
- Average plaquette for $N_f = 4$ and $N_f = 12$ at different volumes.

$SU(3)$ with N_f= 4 and 12 (with Yuzhi Liu and Don Sinclair)



- $< \bar{\Psi}\Psi >$ for $N_f = 4$ and $N_f = 12$ at different volumes.

$SU(3)$ with N_f= 4 and 12 (with Yuzhi Liu and Don Sinclair)



- Fisher's zeros for $N_f = 4$ and $N_f = 12$ at different volumes.

B-physics beyond the standard model

- $B_s \to \mu^+\mu^-$
- $B \to D\tau\nu$
- $\bar{B}_s \to K^+\mu^-\bar{\nu}$

Example of Weak Matrix Element: BR($B_s \to \mu^+\mu^-$)

Daping Du (Fudan, U. of Iowa, Fermilab, U. of Illinois) et al. PRD 85
The BR($B_s \to \mu^+\mu^-$) is very small: $(3.6 \pm 0.4) \times 10^{-9}$. An observed discrepancy would open a window on possible new physics.

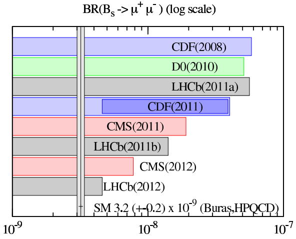


FIG. 1. Comparison of the (most recent) measurements from CDF [8, 11], DØ [9], CMS [12, 14], and LHC*b* [10, 13, 15] with the SM prediction [1, 2] shown as a vertical band. The filled bars show the measured bounds of the branching ratio with a 95% confidence. In the fourth bar, the inner box shows the two-sided 90% bound from CDF [11]. Two results from the LHC*b* in 2011 are distinguished as "2011a" [10] and "2011b" [13].

BR($B_s \to \mu^+\mu^-$)

At LHCb, the branching ratio are obtained by using comparison with other normalization channels like $B_u^+ \to J/\psi K^+$ or $B_d^0 \to K^+\pi^-$ in the following way:

$$BR(B_s^0 \to \mu^+\mu^-) = BR(B_q \to X)\frac{f_q}{f_s}\frac{\epsilon_X}{\epsilon_{\mu\mu}}\frac{N_{\mu\mu}}{N_X}$$

$$\frac{f_d}{f_s} = 12.88\frac{\tau_{B_d}}{\tau_{B_s}}\frac{\epsilon_{D_s\pi}}{\epsilon_{D_dK}}\frac{f_0^{(s)}(m_\pi^2)}{f_0^{(d)}(m_K^2)}\frac{a_1(D\pi)}{a_1(DK)}\frac{N_{D_dK}}{N_{D_s\pi}}$$

$$f_0^{(s)}(m_\pi^2)/f_0^{(d)}(m_K^2) = 1.046(44)_{\text{stat.}}(15)_{\text{syst.}}$$

using MILC ensembles of gauge configurations with 2+1 flavors of sea quarks, with an improved staggered action, on (at best) $28^3 \times 96$ lattices with lattice spacing $a \simeq 0.1$ fermi (so $L_{Phys.} \simeq 0.3hc/m_\pi c^2$)

$B \to D\tau\nu$ (Daping Du)

This work also led to a second, serendipitous paper on a hint of new physics. The contribution of scalar form factors to the semileptonic decay $B \to D\tau\nu$ in the standard and 2-Higgs models provide a possible interpretation for the recent discrepancy found at BaBar. More detail can be found in a paper that has just been published as a highlighted PRL.

$B \to D\tau\nu$ (Daping Du)

Motivation: Semileptonic Decay $B \to D\tau\bar{\nu}$

11 ‡ 36

- The ratio

$$R(D) = \frac{Br(B \to D\tau^- \bar{\nu}_\tau)}{Br(B \to D\ell^- \bar{\nu}_e)}$$

  - Large cancellation of experimental and theoretical systematic uncertainties.
  - $H^\pm$ only enters in $BR(B \to D\tau\nu)$.

- ***New*** BaBar results *1205.5442v1*

Decay	N_{sig}	$\mathcal{R}(D^{(*)})$	$\mathcal{B}(B \to D^{(*)}\tau\nu)$ (%)	Σ_{stat}	Σ_{tot}
$B^- \to D^0\tau^-\overline{\nu}_\tau$	314 ± 60	$0.429 \pm 0.082 \pm 0.052$	$0.99 \pm 0.19 \pm 0.13$	5.5	4.7
$\overline{B}^0 \to D^+\tau^-\overline{\nu}_\tau$	177 ± 31	$0.469 \pm 0.084 \pm 0.053$	$1.01 \pm 0.18 \pm 0.12$	6.1	5.2
$\overline{B} \to D\tau^-\overline{\nu}_\tau$	489 ± 63	$0.440 \pm 0.058 \pm 0.042$	$1.02 \pm 0.13 \pm 0.11$	8.4	6.8

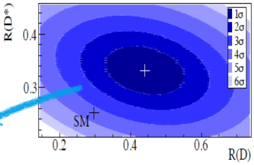
Vera Luth, talk at FPCP2012

	R(D)	R(D*)
BABAR	0.440 ± 0.071	0.332 ± 0.029
SM	0.297 ± 0.017	0.252 ± 0.003
Difference	2.0 σ	2.7 σ

Exp constrained FFs
Phys.Rev.D78:014003

3.8 σ

BABAR

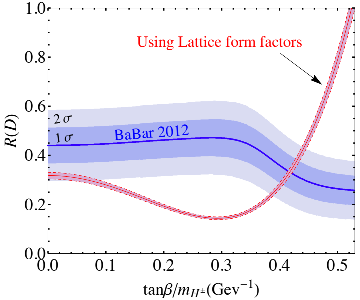


Application: $B \to D\tau\nu$, 2HDM II?

34 ‡ 36

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{cb}|^2}{48\pi^3}\frac{|\mathbf{p}_D|}{q^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2\left[|\mathbf{p}_D|^2|f_+|^2(m_\ell^2+2q^2)+\frac{3}{4}\frac{(M_B^2-M_D^2)^2}{M_B^2}|f_0|^2 m_\ell^2\right]$$

$$\left|1-\frac{q^2}{1-\frac{m_c}{m_b}}\frac{\tan^2\beta}{M_{H^\pm}^2}\right|^2$$



$\bar{B}_s \to K^+\mu^-\bar{\nu}$ (Y. Liu URA)

The goal for the stay at Fermilab is to calculate the form factors for the $\bar{B}_s \to K^+\mu^-\bar{\nu}$ decay mode by using publicly available gauge configurations and techniques developed by the Fermilab/MILC collaboration. This project is intended to provide a chance to predict the shape and normalization before the currently running LHCb experiment. It will eventually lead to a new way to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$. From a long term point of view, precise measurement of the $|V_{ub}|$ is essential to the Fermilab's potential kaon program, such as ORKA and prospects of Project X.

There have been DOE supported visits to Fermilab before the beginning of the URA fellowship in August 2012. The plan is to visit Fermilab about once a month for a period of three to four days each. The request for a seven-months on site stay supported by the URA from mid-August 2012 to mid-March 2013 was approved. Another URA proposal for March to July 2013 was approved too.

Future URA

We plan to involve one or two graduate students with URA support at Fermilab during the Academic Years 2013-2014 and 2014-2015. There are projects in lepton-nucleon scattering relevant to Project X and that would also allow us to take advantage of Prof. Reno expertise.
The low energy neutrino scattering or the conversion process $\mu N \to eN$ that can be approached with a combination of effective field theory aspects and lattice simulations.
There are also some interesting calculations that are needed for the proposed Project X kaon physics program such as the long-distance contributions to the rare kaon decay $K \to \pi l^+ l^-$. Currently the Standard Model estimate relies on Chiral Perturbation Theory and has large uncertainties, which is why this measurement doesn't have as much discovery potential as the "golden mode" $K \to \pi\nu\bar{\nu}$ for which long-distance contributions are GIM suppressed and the Standard Model prediction is quite precise.

New methods in lattice field theory focused toward 3D and 4D $U(1)$ lattice gauge theory

- improved perturbative methods
- RG methods
- New: Tensor Network methods

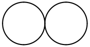
Goals (Haiyuan Zou)

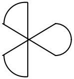
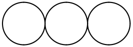
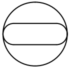
- Using improved perturbative methods to obtain the larger order weak and strong coupling expansion and obtain the nonperturbative contributions. TRG is our choice.
- Starting from less complicated models with low dimensions. E.g. 1-d and 2-d O(2) models.
- 3d and 4d U(1) cases later.

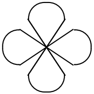
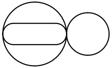
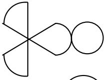
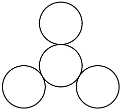
We calculate the weak coupling expansions ($g^2 \sim 1/\beta$) of $L-$link 1-d $O(2)$ model with periodic boundary conditions.
(I) The standard way is using Feynman rules:

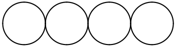
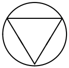
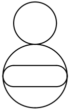
$$E_F \equiv \ln Z[\beta] = \text{const} + b_0 \ln \beta + b_1/\beta + b_2/\beta^2 + b_3/\beta^3 + O(1/\beta^4),$$

with

$b_0 = -\frac{1}{2}(L^d - 1)$, $\quad b_1 = \frac{1}{8}$  ,

$b_2 = -\frac{1}{48}$  $+ \frac{1}{16}$  $+ \frac{1}{48}$  ,

$b_3 = \frac{1}{384}$  $- \frac{1}{48}$  $- \frac{1}{32}$  $+ \frac{1}{48}$ 

$+ \frac{1}{32}$  $+ \frac{1}{48}$  $+ \frac{1}{24}$  .

1-d $O(2)$ model

All the diagrams can be evaluated exactly. The partition function is:

$$Z_{p.b.c}[\beta] \equiv e^{E_F} = \text{const} \cdot \beta^{-(L-1)/2}(a_0 + a_1/\beta + a_2/\beta^2 + a_3/\beta^3 + O(1/\beta^4))$$

in which

$$a_0 = 1, \quad a_1 = \frac{L}{8}\left(1 - \frac{1}{L}\right)^2, \; a_2 = \frac{L^2}{128}\left(1 + \frac{4}{L} - \frac{62}{3L^2} + \frac{28}{L^3} - \frac{37}{3L^4}\right),$$

$$a_3 = \frac{L^3}{3072}\left(1 + \frac{18}{L} + \frac{87}{L^2} - \frac{732}{L^3} + \frac{1735}{L^4} - \frac{1782}{L^5} + \frac{673}{L^6}\right).$$

(II) We have an alternative way by using the asympototic behavior at large β:

$$\frac{I_n(\beta)}{I_0(\beta)} \approx \exp(-\frac{n^2}{2\beta})(1 + f(n, O(\frac{1}{\beta^2})))$$

in which

$$f(n, O(\frac{1}{\beta^2})) = -\frac{n^2}{4\beta^2} + \frac{-13n^2 + 2n^4}{48\beta^3} + \frac{-14n^2 + 5n^4}{32\beta^4} + \cdots$$

1-d $O(2)$ model

By increasing the order of $f(n, O(\frac{1}{\beta}))$, we can calculate high orders of $1/\beta$. E.g., we get the coefficients of the system with $L = 36$ up to order 12.

$$Z[\beta]_{L=36} = \frac{1}{786432\sqrt{2}\beta^{35/2}\pi^{35/2}} \sum_{n=0}^{\infty} a_n \beta^{-n}$$

The coefficients a_n are listed in the table:

n	a_n	n	a_n
0	1	6	$\frac{4316626603928844905 5}{24651234519338188 8}$
1	$\frac{1225}{288}$	7	$\frac{29740143866175908609 45}{788839504618822041 6}$
2	$\frac{5521355}{497664}$	8	$\frac{553787005907486065883854 7}{605828739547255327948 8}$
3	$\frac{3379332985}{143327232}$	9	$\frac{36285132234453667579245674153 9}{1413277283615837229038960 64}$
4	$\frac{7587943455281}{165112971264}$	10	$\frac{688330862660182448514514762309975}{81404771536272224392644132864}$
5	$\frac{12571105207373279}{142657607172096}$	11	$\frac{2286088231017615007596833842882068655}{7033372260733920187524453079449 6}$

Effect of boundary conditions in 1D $O(2)$ (with Haiyuan Zou)

At finite volume, the nonperturbative parts of the average energy are very different for open and periodic boundary conditions

$$\begin{aligned} |(E - E_{PT})/E| &\propto \mathrm{e}^{-2\beta}(\textit{open b.c.}) \\ &\propto \mathrm{e}^{-\beta E_v}(\textit{periodic b.c.}) \end{aligned}$$

where E_v is the energy of the periodic solution of the classical equation of motion with winding number 1 and E_{PT} is the average energy calculated as a power series in $1/\beta$ using conventional Feynman diagrams. For $D \geq 2$ such calculation would require stochastic methods (DMC (see Svistunov and Deng's talks), SPT, ...).

Effect of boundary conditions in 1D $O(2)$ (with Haiyuan Zou)

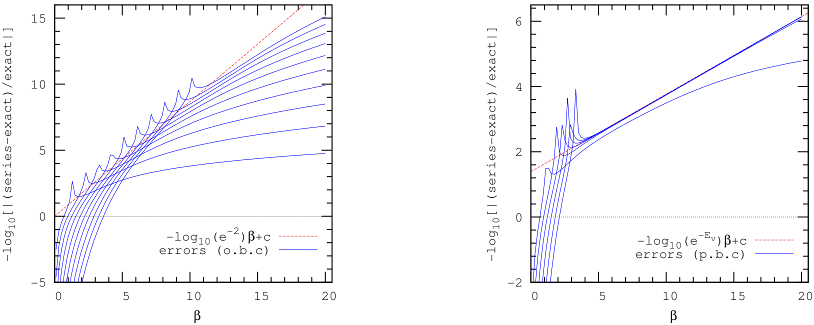


Figure: o.b.c(Left): Errors of the average energy series with order 2,4,...,20; p.b.c($L = 36$)(Right): Errors of the average energy series with order 2,4,...,12.

Comparison of Hadamard series (with Haiyuan Zou)

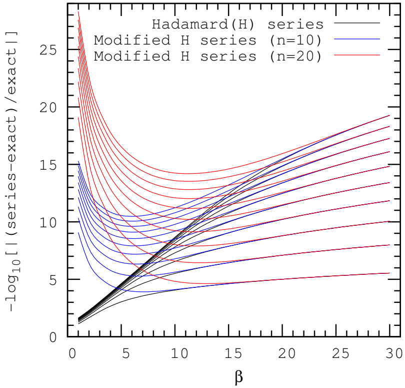


Figure: Errors of different series with order 2,4,...,20.Black:Hadamard; Blue:modified Hadamard($n = 10$); Red: modified Hadamard($n = 20$)

1D O(2) with $L = 4, 8, 16, 32$ (with Haiyuan Zou)

Haiyuan Zou (Shandong U., U. of Iowa)

The zeros are very different for open (o.b.c) and periodic boundary conditions (p.b.c):

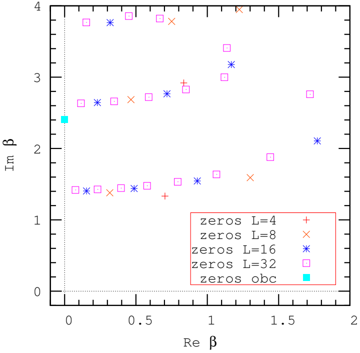
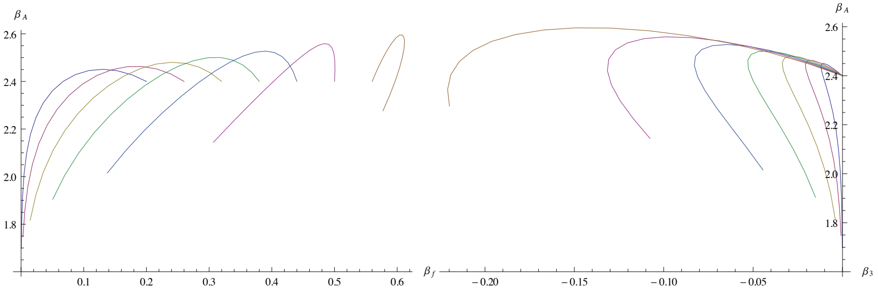


Figure: Zeros of partition function (p.b.c) with different volumes and zeros of partition function (o.b.c)

$SU(2)$ with $\beta_{Adjoint}, \beta_{3/2}, \ldots$ (with J. Unmuth-Yockey)

The MK approximation allows us to deal with nonlinear aspects of the RG flows. In the Migdal-Kadanoff approximation, RG flows can go around phase boundaries (not shown).



Two-lattice matching

Starting with a theory on a lattice with $(2L)^D$ sites and a set of couplings $\{\beta_i\}$, blockspinning provides a new theory on a lattice with L^Dsites and new effective couplings $\{\beta'_i\}$

We have the exact identity relating a $2Mx2M$ Wilson loop W on the original lattice to a MxM Wison loop on the coarse lattice:

$$< W_{2Mx2M} >_{2L,\{\beta_i\}} = < W_{MxM} >_{L,\{\beta'_i\}}$$

If you can calculate the Wilson loops numerically using MC, you can fine-tune the $\{\beta'_i\}$ on the coarse lattice in order to match the values on the fine lattice. This is done with a finite number of couplings (often one) and provides an approximate discrete flow of $\{\beta_i\}$.

For spin models, the matching can be applied between correlations of blocks of size $2M$ and M

2-lattice matching using Migdal-Kadanoff (with Alan Denbleyker and Judah Unmuth)

Is the MK approximation reliable? The MC calculation of $2Mx2M$ Wilson loops for a $(2L)^4$ lattice and the MxM Wilson loop on a L^D lattice with effective couplings obtained by the MK recursion show that the matching is not very accurate. LPA improvement is needed!

Volume	b	β_F	β_A	$\beta_{3/2}$	β_2	P_{size}	$\langle P \rangle$	σ
8^4		2.40000	0.00000			$2x2$	0.7766	0.00672
4^4	2	0.955274	-0.0496152	0.003759328	-0.000310275	1x1	0.7710	0.01226
8^4		2.40000	0.00000			$4x4$	0.9009	0.09007
4^4	2	0.955274	-0.0496152	0.003759328	-0.000310275	$2x2$	0.9973	0.01283
8^4		4.80000	0.00000			$2x2$	0.4016	0.00369
4^4	2	4.47578	-0.728286	0.188086	0.055336	1x1	0.2225	0.00655
8^4		4.80000	0.00000			$4x4$	0.5670	0.12841
4^4	2	4.47578	-0.728286	0.188086	0.055336	$2x2$	0.5144	0.01799

- M-K RG relies upon bond moving, and re-summing
- First every other bond in one dimension is shifted over one
- Continue this process for each dimension
- Now sum over the “weakened” sites
- The result is a lattice with modified coupling and double the lattice spacing.

- Migdal and separately Kadanoff have devised a coarse graining method for lattice renormalization.

$$\int dV\ \chi_r(UV)\chi_s(W^\dagger V) = \frac{\delta_{rs}}{d_r}\chi_r(UW) \tag{1}$$

$$e^{-S_p(U,a)} = \sum_r F_r(a) d_r \chi_r(U) \tag{2}$$

$$F_r(a) = \frac{1}{d_r}\int dU\ e^{-S_p(U,a)}\chi_r^*(U) \tag{3}$$

$$\boxed{e^{-S_p(U,\lambda a)} = \left[\sum_r F_r(a)^{\lambda^2} d_r \chi_r(U)\right]^{\lambda^{d-2}}} \tag{4}$$

- To make the recursion formulae we insert **??** into **??**.

$$e^{-S_p(V,\lambda a)} = \left[\sum_r \left(\frac{1}{d_r} \int dU \, e^{-S_p(U,a)} \chi_r^*(U) \right)^{\lambda^2} d_r \chi_r(V) \right]^{\lambda^{d-2}} \quad (5)$$

- Using this, we can start with some inital action, $S_0(U, a)$ and preform recursions for subsequent actions $S(U, \lambda a)$.
- Looking at the flows of β in multiple dimensions (representations), there appears to be a fixed point at zero.
- The flows also appear to avoid the phase transition line.
- After each iteration, M-K RG guarantees a supremum for the partition function.

The Problem of Confinement for Pure Gauge Yang-Mills

Judah Unmuth-Yockey

- An improved M-K RG scheme has been used in an attempt at proving confinement (e.g. see Tombulis (2007)).
- However, currently such a proof implies that U(1) is confining for all values of β.
- We are looking into ways to improve MK using TNRG to gain insight in proofs of confinement.
- Improvments have been found by using “the two-state approximation”, where the model is mapped into itself, similar to M-K RG, however using tensor networks.

Complex RG flows using two lattice matching

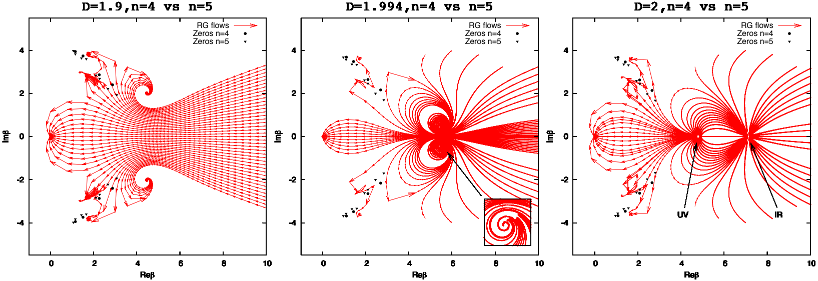


Figure: Complex RG flows for "log-deformed" hierarchical models. See Y. Liu, YM, H. Zou, arXiv:1112.3119, POS Lattice 2011 246.

Continuum limit of a discrete RG transformation

The Hierarchical Model recursion formula can be extended for an arbitrary scale factor b. The Fourier transform of the recursion formula

$$R_{n+1}(k) = C_{n+1}\, \mathrm{e}^{-\frac{1}{2}\beta\frac{\partial^2}{\partial k^2}} \left(R_n(\sqrt{c/4}\, k)\right)^2 ,$$

becomes ($2 = b^D$)

$$R_{n+1}(k) = C_{n+1}\, \mathrm{e}^{-\frac{1}{2}\beta\frac{\partial^2}{\partial k^2}} \left(R_n(b^{-(D+2)/2}\, k)\right)^{b^D} ,$$

In the limit $b \to 1$, and after some transformations, one obtains the WP equation

$$\frac{\partial V}{\partial t} = DV + (1 - \frac{D}{2})\phi\frac{\partial V}{\partial \phi} - (\frac{\partial V}{\partial \phi})^2 + \frac{\partial^2 V}{\partial \phi^2}$$

known to be equivalent to the optimal ERG LPA (see YM J. Phys. A 40 R39-102 for Refs.)

The optimal *ERGLPA*

Using the basic ERG equation for a $N = 1$ scalar in D dimensions

$$\partial_t \Gamma = \frac{1}{2} Tr \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

with the LPA ansatz $\Gamma_k = \int d^D x (U_k + \frac{1}{2}(\partial\phi)^2)$ we obtain

$$\partial_t u = -Du + (D-2)\rho u' + C \int_0^\infty dy y^{D/2} \frac{\partial_t r}{y(1+r) + u' + 2\rho u''}$$

where u, ρ and r are suitably rescaled versions of U, ϕ^2 and R. Using Litim's optimal cutoff function $r = (1/y - 1)\theta(y-1)$ and more rescalings, one obtains the canonical form

$$\partial_t u = -Du + (D-2)\rho u' + \frac{1}{1 + u' + 2\rho u''}$$

Small difference between the optimal and HM exponents

A very nice feature of the LPAs is that they allow **very accurate** calculations.

The exponents for the HM and optimal LPA are very close:
$\nu_{HM} = 0.649570365$
$\nu_{opt.} = 0.649561773$
(Litim; Bervillier, Juttner and Litim)

This is far from the conventional Ising universality class:
$\nu_{Ising3} \simeq 0.6304$

The exponents for the HM with $b^D = 3, 4, 5, ...$ are also close and can be calculated accurately (with Y. Liu and B. Oktay). In the following we call this series of exponents the “discrete series".

The calculations for b^D noninteger are numerically unstable for reasons that can be identified by considering the calculation at $b^D = 2 + \epsilon$.

Attempt to fit the discrete series with ERG LPAs

We consider the family of LPAs

$$\partial_t u = -3u + \rho u' + \frac{1}{4\pi^2}\int_0^\infty dy \frac{-y^{\frac{5}{2}} r'(y)}{y(1+r) + u' + 2\rho u''}$$

with cutoff functions $r(y) = B(\frac{1}{y} - 1)\theta(1-y)$ ($B = 1$ is optimal, see Litim PRD 76 105001)

Expanding in $B - 1$, truncating, and rescaling the first two corrections independently, the RHS becomes

$$-3u + \rho u' + \frac{1}{1+w} + \epsilon_1 \frac{1}{(1+w)^2} + \epsilon_2 \frac{1}{(1+w)^3}$$

with $w = u' + 2\rho u''$ and where ϵ_1 and ϵ_2 are now small independent parameters. In the following, we get a series of ν and ω by changing ϵ_1 from -0.03 to 0.02 with step 0.005 and ϵ_2 from 0 to 0.02 with step 0.0005 (Yuzhi Liu)

ν and ω for LPA(ϵ_1, ϵ_2) and HM $b^D = 2,\ 3,\ \cdots 8$ (Y. Liu)

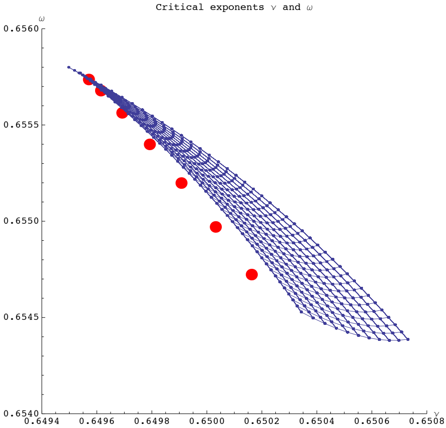


Figure: ω versus ν for the HM $b^D = 2,\ 3,\ \cdots 8$ (red) and LPA(ϵ_1, ϵ_2) (blue).

Decimation

In this approach, we repeatedly integrate over a fraction of the ϕ_x. This works in $D = 1$ or by "bond sliding" approximations (Migdal).

Example: 1-D Ising, 2^n sites and periodic boundary conditions:

$Z = \sum_{\{\sigma_i=\pm 1\}} \mathrm{e}^{\beta \sum_j \sigma_j \sigma_{j+1}}$

Using $\mathrm{e}^{\beta\sigma} = \cosh\beta + \sinh\beta\sigma$ (for $\sigma = \pm 1$)

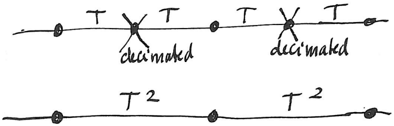
$$\begin{aligned}\sum_{\{\sigma_1=\pm 1\}} & (\cosh\beta + \sinh\beta\sigma_0\sigma_1)(\cosh\beta + \sinh\beta\sigma_1\sigma_2) \\ = & \ 2(\cosh^2\beta + \sinh^2\beta\ \sigma_0\sigma_2)\end{aligned}$$

Factoring out the cosh 's we get $\tanh\beta' = \tanh^2\beta$

In arbitrary dimension, it is possible to use this representation to write the partition function as a sum over link configurations. The links can take the values 0 and 1 with "current conservation modulo 2".

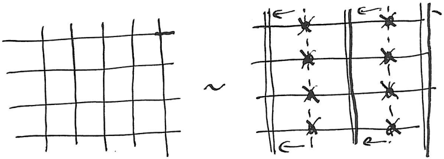
Tensor Network Formulation

Decimation in 1D : $\mathrm{tr}\, T^L = \mathrm{tr}(T^2)^{\frac{L}{2}}$



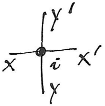
(integration over every other site)

2D → M-K Migdal-Kad.



2D → Tensor Network : replace sum over site variables by sum over link variables

$$Z = \mathrm{Tr}\left(\prod_i T^{(i)}_{xx'yy'} \right)$$



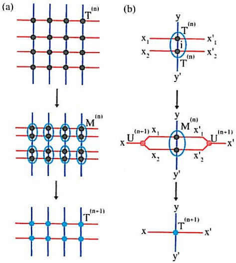


FIG. 1: (a) A HOTRG contraction of the tensor network state along the y axis on the square lattice. (b) Steps of contraction and renormalization of two local tensors. The initial tensor $T^{(0)} = T$.

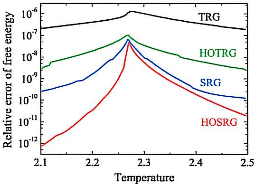
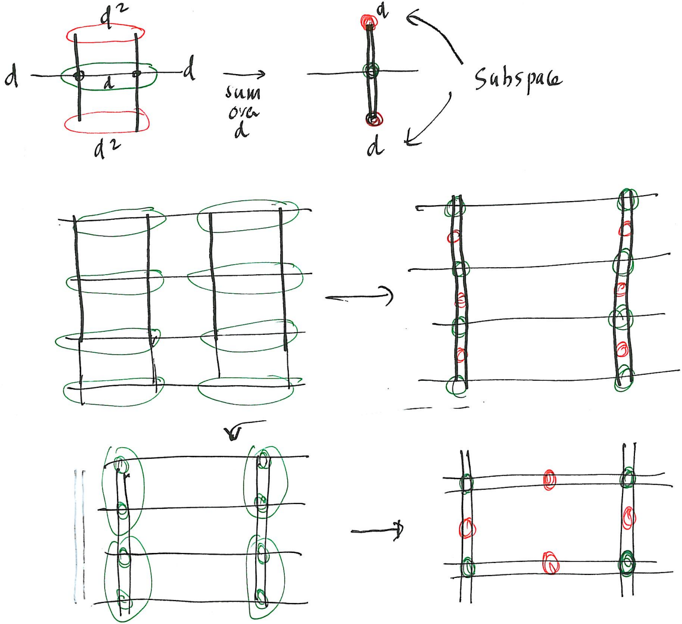


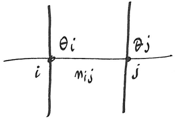
FIG. 4: (color online) Comparison of the relative errors of free energy with respect to the exact results for the 2D Ising model obtained by various methods with $D = 24$. The critical temperature $T_c = 2/\ln(1+\sqrt{2})$.

From Z.Y. Xie, J. Chen, M.P.Qin, J.W. Zhu, L.P. Yang and Tao Xiang PRB 86 045139 (2012)

Tensor Network Coarse Graining



O(2) in 2D



$$e^{\beta\, \vec{S}_i \cdot \vec{S}_j} = e^{\beta \cos(\theta_i - \theta_j)}$$

$$= \sum I_{n_{ij}}(\beta)\, e^{i n_{ij} (\theta_i - \theta_j)}$$

$$= \langle \theta_i | \Lambda | \theta_j \rangle$$

$$= \sum_{n_{ij}} \langle \theta_i | n_{ij} \rangle\, \lambda_{n_{ij}} \langle n_{ij} | \theta_j \rangle$$

$$Z = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \prod_{\langle ij \rangle} \sum_{n_{ij}} I_{n_{ij}}(\beta)\, e^{i n_{ij} (\theta_i - \theta_j)}$$

$$= \sum_{\{n_{ij}\}} \prod_i \left[\delta_{n_{in}, n_{out}} \prod_{\substack{j \\ \langle ji \rangle}} \sqrt{I_{n_{ij}}(\beta)} \right]$$

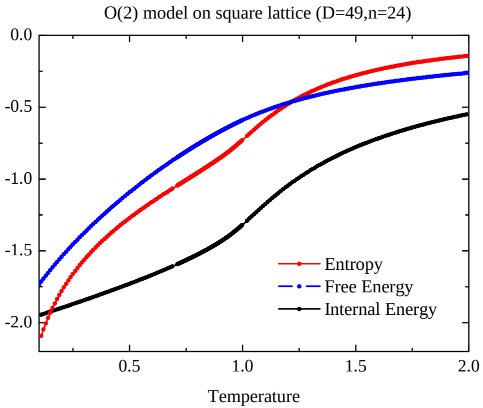


Figure: Free energy, entropy and average energy for the $O(2)$ model, with Tao Xiang, Zhiyuan Xie and Yuzhi Liu.

Specific Heat for $O(2)$

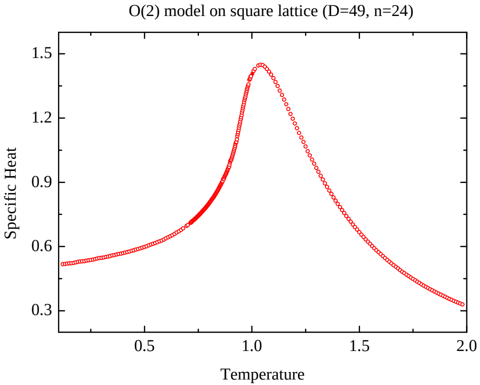
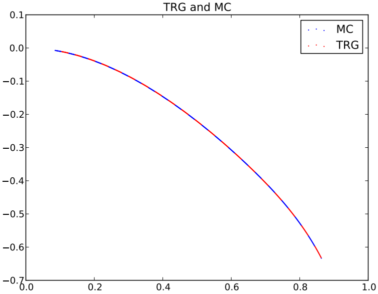


Figure: Specific heat for the $O(2)$ model, with Tao Xiang, Zhiyuan Xie and Yuzhi Liu.

Difference with MC (Alan Denbleyker)



Approximate recursions for tensor renormalization

- We proposed approximate recursion formulas for Ising models based on two-state truncations of the Tensor Renormalization Group (TRG) approach of classical lattice models. In two dimensions,we consider the cases of an isotropic blocking (as in the Migdal recursion) and an anisotropic blocking (as in the Kadanoff version) with the two state projection based on a higher order singular value decomposition (HOSVD) used by T. Xiang et al. We also consider a projection based on a 2 by 2 transfer matrix.
- The transformation can be expressed as a map with 3 and 4 parameters in the isotropic and anisotropic cases respectively.
- Linear analysis near the nontrivial fixed point yields $\nu = 0.987$, 0.964 and 0.993 for the three maps respectively, which is much closer to the exact value 1 than 1.338 obtained in the Migdal and Kadanoff approximation.
- The method can be applied to other models (3D Ising and models with lattice fermions).

Improvement of the HM LPA: restoration of translation symmetry?

The LPA associated with block spinning procedure requires that we isolate the block from its environment while performing the integrations. This typically create “walls" that break translational invariance but allow to do the calculation. This generates hierarchical basis for the field configurations that can be analyzed with some “tree symmetry".

I propose to try to improve the LPA by breaking the tree symmetry little by little while restoring the translational invariance

The success of the tensor network formulation can be explained by the fact that the states are attached to the links. They are either inside the block and summed over or piercing the boundary and kept free.

LGT calculations on Optical Lattice?

- The possibility of trapping polarizable atoms or molecules in a periodic potential created by crossed counterpropagating laser beams has been an area of intense activity in recent years.
- It is now possible to physically build lattice systems where the number of particles and their tunneling between neighbor sites of the lattice can be adjusted experimentally.
- This opens the possibility of engineering experimental setups that mimic lattice Hamiltonians used by theorists (e.g. the Bose-Hubbard model) and to follow their real time evolution.
- The versatile technology of cold atoms confined in optical lattices allows the creation of a vast number of lattice geometries and interactions.