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General Motivations

- Large distance effects in QCD (crucial for masses, decay constants and mixing angles) can only be handled with Monte Carlo simulations, but getting rid of lattice effects and finite size effects is costly (the "Berlin-Wall"):

$$\text{Cost} \propto (\text{Lattice size})^5\ (\text{Lattice spacing})^{-7}$$

- Finite Size Scaling (FSS) is essential to extrapolate the properties of the QCD finite temperature phase transition to infinite volume using numerical data on small lattices. Nonlinear effects in FSS can be important at sufficiently large volume. It is difficult to distinguish these effects from the statistical errors. There is not much numerical evidence that the predictions of the linear FSS are well obeyed at large volume.

- The short distance effects in QCD can be described by perturbation theory (asymptotic freedom), however QCD series diverge fast. For the hadronic width of the Z^0, the term of order α_s^3 is more than 60 percent of the term of order α_s^2 and contributes to one part in 1,000 to the total width (a typical experimental error at LEP). Next Leading 0rder (NLO) and NNLO corrections can be significant for processes relevant for the LHC (for instance, $pp \rightarrow Z+4$ jets). Nobody knows if the next order will improve or worsen the accuracy! (And such calculations can take years!)

- The experimental error bars of the anomalous magnetic moment of the the muon a_μ have been shrinking. We are reaching the limit of perturbative accuracy in theoretical estimates. Precision tests may become a major source of information regarding new laws of nature in the long term.

- Perturbation theory imposes stability and triviality bounds on the Higgs mass m_H. If we believe these perturbative bounds, m_H should be in a small window near $m_H = 175\,GeV$, unless there is new physics at a scale Λ (see T. Hambye and K. Riesselmann, Phys. Rev. D 55, 7255, 1997). However, these bounds probably reflect the failure of perturbation theory (K. Holland, Lattice 2004)

Summary

- There is an urgent need to develop new analytical methods based on weak coupling perturbative expansions, but modified in such way that they can be extended beyond the range of conventional methods.

- The main problem of perturbation theory is that it cannot successfully handle large quantum fluctuations (the large field contributions in the path integral). The large field configurations become important in the crossover region between weak and strong coupling (at zero temperature) and near the critical coupling (at finite temperature).

- The renormalization group method (including FSS) is essential to make calculations in the crossover region (at zero temperature) and to understand the finite temperature phase transition.

Recent Accomplishments

- Understanding of the large order behavior of the perturbative expansion of the average plaquette (the lattice analog of the gluon condensate) in quenched QCD. The apparent singularities are off the real axis (in the complex $\beta = 2N/g^2$ plane; no third-order phase transition). Mean field theory explains the series up to order 20. Infrared renormalons probably dominate beyond order 25.

- Semi-classical parametrizations of the non-perturbative part of the plaquette and the β-function in quenched QCD. This suggests a semi-classical approach of the lattice scaling.

- Universal behavior of the modified perturbative coefficients (calculated numerically with a large field cutoff) in quantum mechanics and for the hierarchical model.

- Approximate equivalence between the renormalization group transformation of the hierarchical model and Polchinski's renormalization group equation in the local potential approximation (exponents differ by less than 10^{-5}).

- Numerical calculation of critical effective potentials and understanding of their complex singularities for scalar and gauge models.

- New methods to locate zeros of the partition function.

- New method to control nonlinear effects in Finite Size Scaling (FSS).

Recent Publications

1. Y. MEURICE, *How to control nonlinear effects in Binder cumulants*, preprint in first draft stage (content discussed in coming slides).

2. A. DENBLEYKER, D. DU, Y. MEURICE, and A. VELYTSKY, *Fisher's zeros in $SU(2)$ and $SU(3)$ gluodynamics*, preprint in progress(content discussed in coming slides).

3. A. DENBLEYKER, D. DU, Y. MEURICE, and A. VELYTSKY, *Fisher's zeros of quasi-Gaussian densities of states*, Phys. Rev. **D76**, in press (2007), [arXiv:0708.0438 [hep-lat]].

4. A. DENBLEYKER, D. DU, Y. MEURICE, and A. VELYTSKY, *Fisher's zeros and perturbative series in gluodynamics*, PoS **LAT2007**269 (2007) [arXiv:0710.5771 [hep-lat]].

5. Y. MEURICE, *Nonlinear Aspects of the Renormalization Group Flows of Dyson's Hierarchical Model*, J.Phys. **A40** R39-102 (2007), [hep-th/0701191]; solicited topical review.

6. Y. MEURICE, *At which order should we truncate perturbative series?*, Published in *Minneapolis 2006, Continuous advances in QCD* 310-316, World Scientific (2007), [hep-th/0608097].

7. Y. MEURICE, *The non-perturbative part of the plaquette in quenched QCD*, Phys. Rev. **D74**, 096005 (2006), [hep-lat/0609005].

Recent Talks:

1. Y. MEURICE, *Fishers Zeros at Zero and Finite Temperature*, QCD in extreme conditions, http://www.lnf.infn.it/conference/xqcd2007/Talks/Meurice.pdf

2. Y. MEURICE, *Fisher's Zeros and Perturbative Series in Gluodynamics*, LATTICE 2007, http://www.physik.uni-regensburg.de/lat07/hevea/meurice.pdf

3. Y. MEURICE, *Convergent multiloop expansions*, LoopFest VI, http://ubpheno.physics.buffalo.edu/%7Edow/loopfest6/meurice.pdf

4. Y. MEURICE, *Convergent Multilooop Expansion*, talk given at the University of Kansas at Lawrence, April 22 2007.

5. Y. MEURICE, *A RG analysis of the interpolation between weak and strong coupling*, 3rd International Conference on the Exact Renormalization Group, Lefkada (Greece), September 2006. http://www.cc.uoa.gr/ papost/Meurice.pdf

6. D. DU, *Fisher Zeros in SU (2) Lattice Gauge Theory*, talk given at the International Summer School on Lattice QCD, Seattle, August 2007.

7. A. DENBLEYKER, *MC Studies of Fisher Zeros in Spin and Gauge Models*, poster presented at the International Summer School on Lattice QCD, Seattle, August 2007 .

Completion of a Review Article on the Renormalization Group

After more than one year of work, the solicited review article "Nonlinear Aspects of the Renormalization Group Flows of Dyson's Hierarchical Model" has been completed, accepted and published as topical review by Jour. of Phys. A. in spring 2007. The article summarizes recent progress in doing numerical renormalization group calculations for scalar models. One of the merit of this review article was to start a dialogue with other authors using different methods where the renormalization group transformation evolves continuously instead of discretely as in our method. We have recently designed methods which can interpolate between the two approaches (with Y. Liu). I have started a detailed study of Finite Size Scaling (FSS) in this model.

Graduate Student Supervision

In recent years, I made a special effort to recruit high quality graduate students. The University has strict requirements regarding the oral competency of graduate students employed as Teaching Assistant beyond the first year. In the past three years, three of our graduate students were unable to pass the test before the end of the first year. I currently supervise three graduated students.

- Daping Du came in fall 2005. He has passed the qualifying exam and the comprehensive exam. He has not passed the T.A. (Teaching Assistant) certification necessary to lead lab sections. He works on the fits of plaquette distribution, saddle point estimates of the Fisher zeros and the density of states in quenched QCD. He has been supported partially as a T.A. (as grader) and partially as a R.A. (Research Assistant).

- Alan Denbleyker came in fall 2006. He works on MC simulations in $SU(2)$ gauge theories with and without adjoint terms and is planning to extend the existing codes for $SU(3)$. He has devised complex zeros searching algorithm. He is the system manager for our cluster and repository. He has been supported as a T.A.

- Yuzhi Liu came in fall 2006. He has passed the qualifying exam. He has not passed the T.A. certification. He works on the comparison between discrete renormalization group methods that we have been using and continuous limits of these methods used by other authors. He has been supported partially as a T.A. and partially as a R.A..

Summer school: Daping DU and Alan Denbleyker, attended the International Summer School on Lattice QCD and its applications, at the University of Washington Seattle, August 8 - 28, 2007 and gave talks.

Computational Facilities

In 2003, we built a 16 node cluster that has allowed us to create large numbers of gauge configurations on lattices with up to 16^4 sites and to work on scalar field theory in various dimensions. Due to several recent hardware failures, this cluster will be phased out.

We have built a new cluster with 8 single CPU nodes with 3.2 GHz Pentium4 processors and Gigabyte motherboards with a build-in fast ethernet card. It is now being reconfigured with Oscar 5.0.

Multiprocessors computers have been build commercially and are low maintenance items. We would like to try 4 CPU units to use for distributed calculations. This type of workstation costs approximately 5,000 dollars.

Work in progress

- New methods to find the zeros of the partition function in the complex β plane (Fisher's zeros) for Lattice Gauge Theory.

- New methods to control the nonlinear effects in FSS for Binder Cumulants at large volume.

Motivations to study the zeros of the partition function

Lattice QCD in its simplest form (quenched with a Wilson's action) is characterized by two regimes: the strong coupling (low β) regime where confinement is obvious and the weak coupling (large β) regime where asymptotic freedom holds. We expect no phase transition on the real β axis, unlike Ising models where the zeros of the partition function pinch the real axis of the complex inverse temperature at β_c at infinite volume. These zeros in the complex β plane are called Fisher's zeros. However, such lattice gauge theories can be seen as "close" to other theories (at non-zero temperature or with a positive adjoint coupling) that have a phase transition. The zero temperature lattice gauge theory with a Wilson action should have Fisher zeros close to the real axis but these zeros should not pinch the real axis in the infinite volume limit. This is a plausible explanation for the unexpected behavior of the weak coupling expansion of the average plaquette P for $SU(3)$. Standard methods of series analysis suggest a singularity on the real axis, namely $P \propto (1/5.74 - 1/\beta)^{1.08}$. This would imply a peak in the second derivative of P with a height increasing with the volume, which we have not seen at zero temperature. The vicinity of the critical point in the fundamental-adjoint plane, suggests the approximate mean field behavior

$$-\partial P/\partial\beta \propto \ln((1/\beta_m - 1/\beta)^2 + \Gamma^2) \ , \tag{1}$$

Fits of the series with such parametric form yield the approximate values $\beta_m \simeq 5.78$ and $\Gamma \simeq 0.006$ (i.e $Im\ \beta \simeq 0.2$). Values of Γ which are too large (too small) would produce modulations of the coefficients (peaks in the derivatives of P) which are not observed. This yields the bounds for $SU(3)$ of $0.001 < \Gamma < 0.01$. This suggests zeros of the partition function in the complex β plane with $0.03 \simeq 0.001\beta_m^2 < Im\beta < 0.01\beta_m^2 \simeq 0.33$.

It is quite common that the difference between a physical quantity and its perturbative expansion is of the form $\exp(-K/g^2)$. For the average plaquette, the issue is obscured by the Fisher zeros and the factorial growth necessary to get an envelope in the accuracy versus coupling at successive order is not reached at the order where the perturbative expansion is available. Larger order extrapolation are necessary. Two models have been considered. These two extrapolations seem consistent with the behavior

$$P(\beta) - P_{pert.}(\beta) \simeq C(a/r_0)^4 \tag{2}$$

with $a(\beta)$ defined with the force scale with $r_0 = 0.5$ fm, and P_{pert} appropriately truncated. For large β this has the desired exponential form. Attempts have been made in the past to relate C to the so-called gluon condensate.

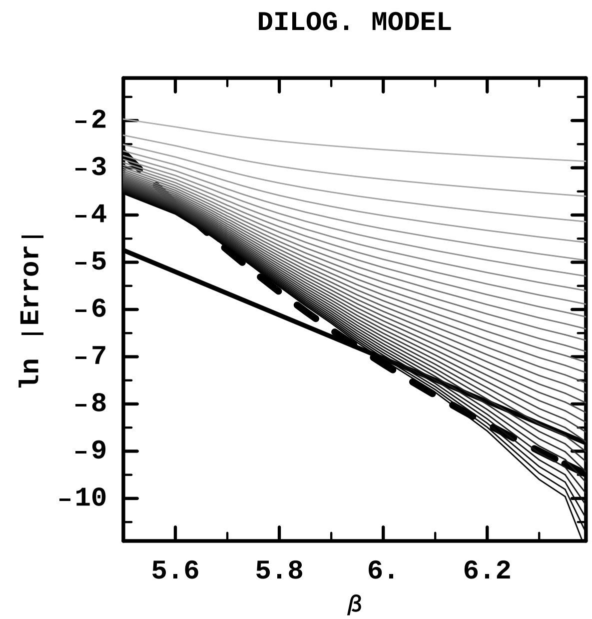

Figure 1: Natural logarithm of the absolute value of the difference between the series and the numerical value for order 1 to 30 for quenched QCD with the dilogarithm model. As the order increases, the curves get darker. The long dash curve is $\ln(0.65\ (a/r_0)^4)$. The solid curve is $\ln(3.1 \times 10^8 \times (\beta)^{204/121-1/2} \mathrm{e}^{-(16\pi^2/33)\beta})$

Searches for zeroes of the partition function

We searched for the zeros of the partition function in the complex β plane by using the reweighting method. The bounds in the $1/\beta$ plane discussed above imply that the the location of the zero closest to the positive real axis in the β plane should have an imaginary part between 0.03 and 0.3 when the volume becomes large enough. If the imaginary part is larger than $K\sqrt{\ln(N_{conf.})/V}$ with $N_{conf.}$ the number of configurations and K a calculable function of the real part of order 1 varying slowly, the determination of the zeroes becomes inaccurate. For small volume (4^4), we were able to establish the existence of a zero clearly within the radii of confidence of various reweighting near $\beta = 5.55 \pm i0.1$ in agreement with results on $4L^3$ lattices. For larger (6^4 and 8^4) lattices, we have not found zeroes clearly within the radii of confidence. Up to now, everything seems consistent with the bound $|Im\beta| > 0.03$.

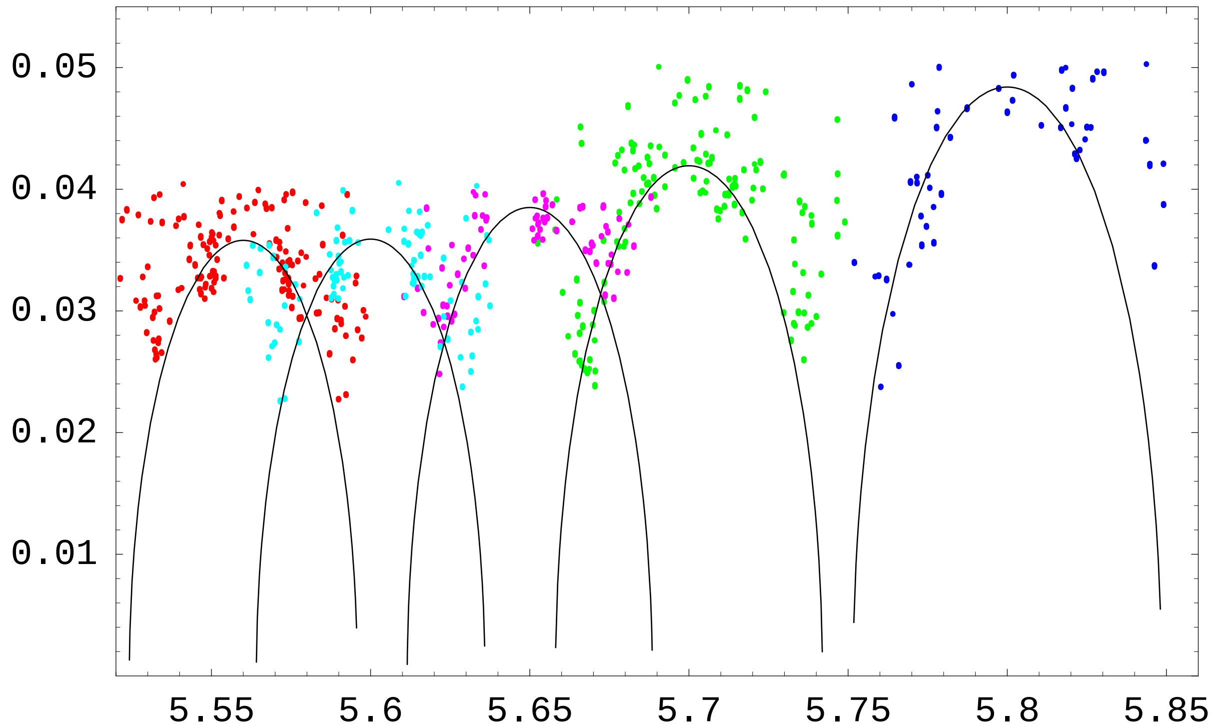

Figure 2: Zeroes of the partition function in the complex β plane for a 8^4 lattice. The dots correspond to distinct bootstraps and the solid lines to the radii of confidence.

Approximate models

The nice regularities of the difference with the Gaussian approximation (for small lattices) for the distribution of action S suggests a parametriztion of the form

$$P(S) \propto \exp(-\lambda_1 S - \lambda_2 S^2 - \lambda_3 S^3 - \lambda_4 S^4) \tag{3}$$

The unknown parameters were determined from the fist four moments using Newton's methods and also by χ^2 minimization. Very good agreement between the two methods was found on 4^4 lattices. This method allows us to go beyond the region of confidence of MC reweighting.

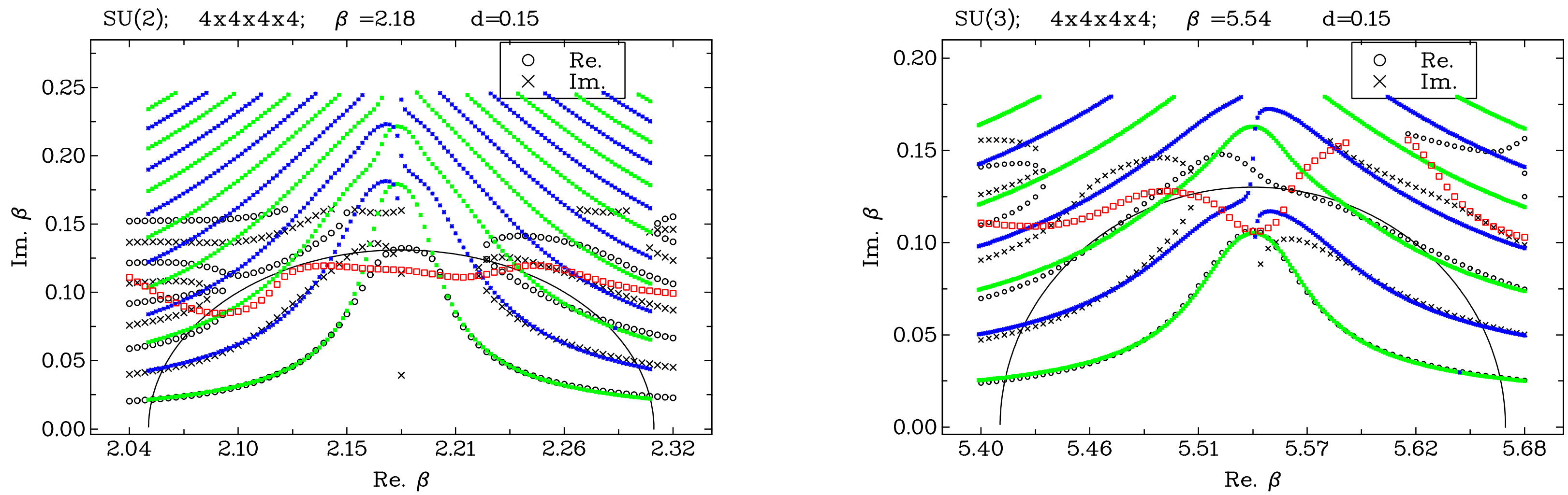

Figure 3: Zeros of the real (crosses) and imaginary (circles) using MC on a 4^4 lattice, for $SU(2)$ at $\beta = 2.18$ and $SU(3)$ at $\beta =$5.54. The smaller dots are the values for the real (green) and imaginary (blue) parts obtained from the 4 parameter model. The MC exclusion region boundary for $d = 0.15$ is represented by boxes (red).

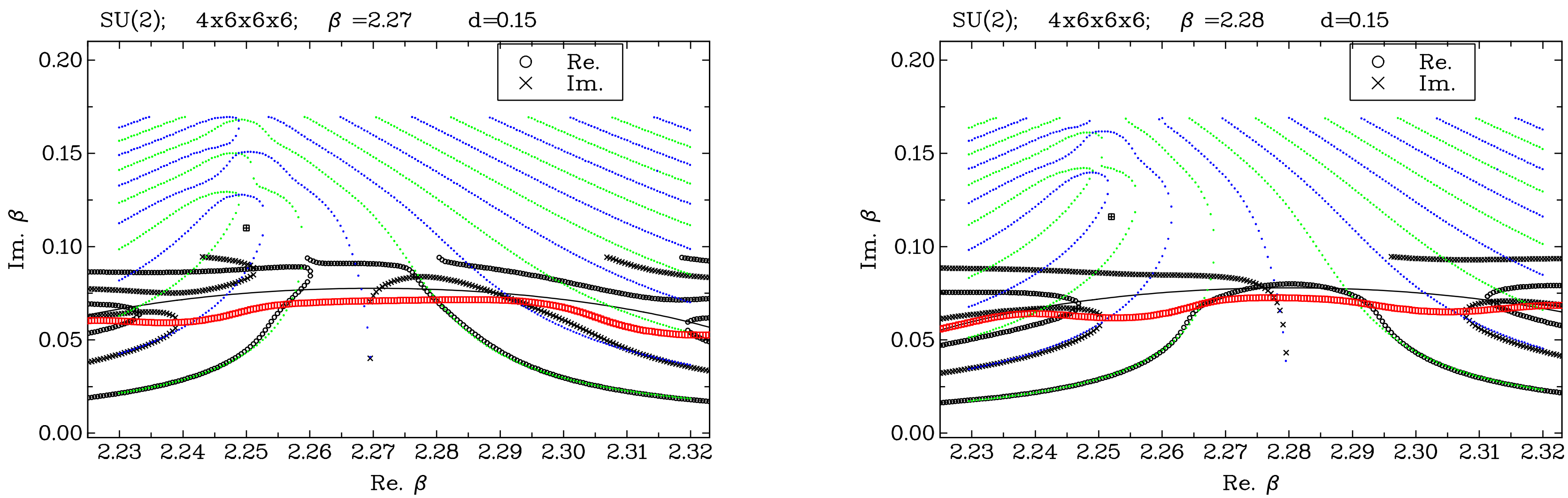

Figure 4: Zeros of the real (crosses) and imaginary (circles) using MC, for $SU(2)$ at $\beta = 2.27$ and 2.28 on a 4×6^3 lattice. The smaller dots are the values for the real (green) and imaginary (blue) parts obtained from the 4 parameter model. The MC confidence region is limited by red boxes. Note the consistency of the intersections for the two reweightings.

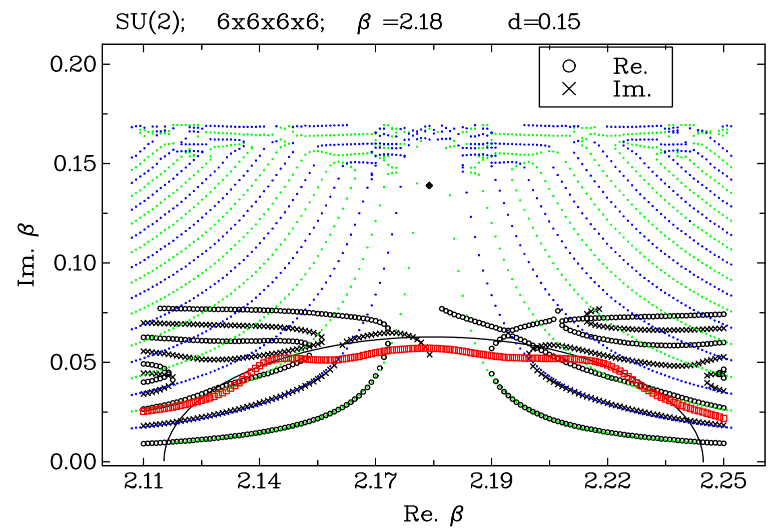

Figure 5: Zeros of the real (crosses) and imaginary (circles) using MC for $SU(2)$ on a 6^4 lattice at $\beta = 2.18$. The small dots are the values for the real (green) and imaginary (blue) parts obtained from the 4 parameter model. D. Du is working on saddle point methods to deal with the noise that appears for $Im\beta > 0.16$.

Recent work on finite size scaling

In the study of the finite temperature phase transition of QCD, it is important to be able to localize precisely β_c and identify the universality class of the transition. This information can be obtained from the study of the so-called Binder cumulant

$$B_4 = \frac{<m_4>}{<m_2>^2}$$

of the Polyakov's loop. In recent studies of Binder cumulants, I noticed that the determination of the intersection of curves at different volumes using linear fits may lead to inaccuracies. I studied the problem for the hierarchical Ising model where the statistical errors are negligible and found that if the β interval scale is too broad, the nonlinear effects are important. These effects can be estimated by studying the collapsed data (B_4 versus $\kappa N^{1/\nu}$ with $\kappa \equiv (\beta - \beta_c)/\beta_c$).

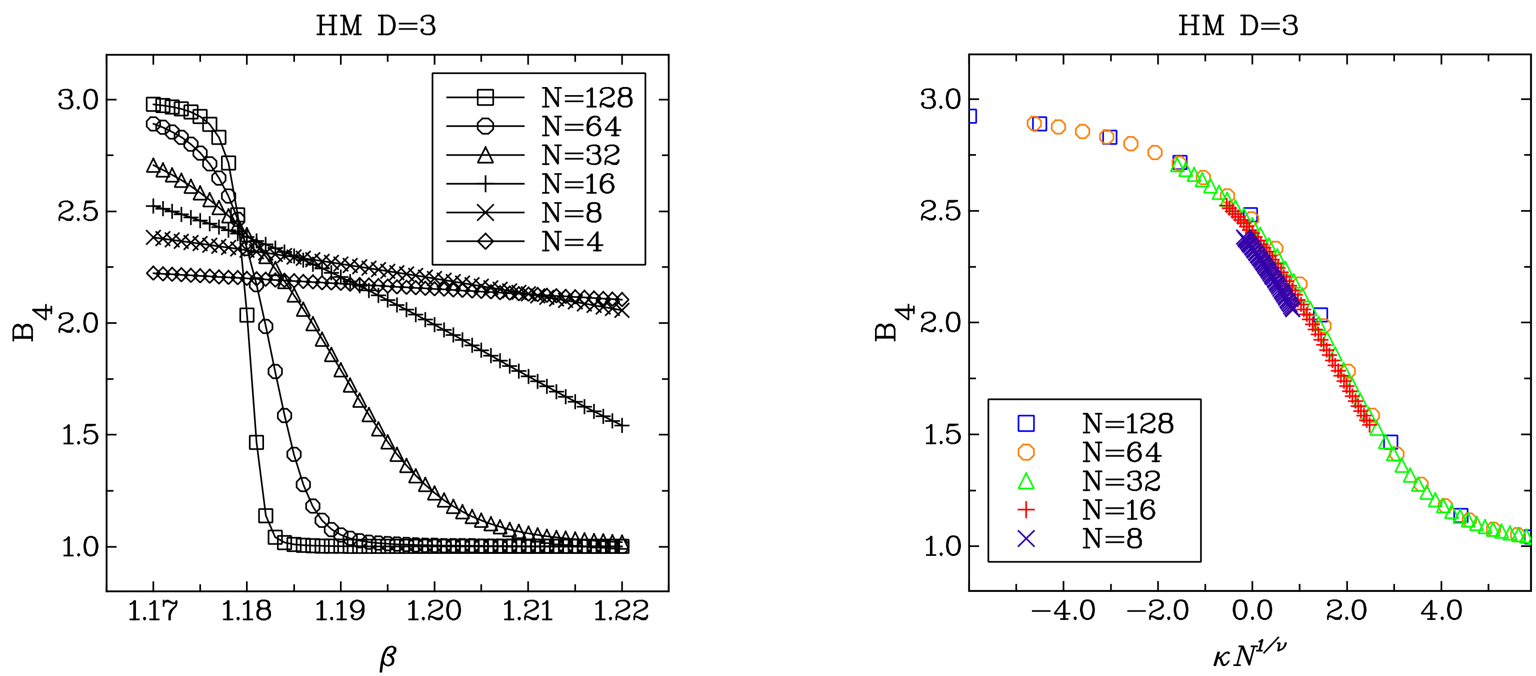

Figure 6: B_4 versus β, and $\kappa N^{1/\nu}$for $N = 4$, 8, 16, 32, 64 and 128 for the Ising hierarchical model.

$$B_4(\beta, N) \simeq B_4(\beta_c, \infty) + f_1 \kappa N^{1/\nu} + f_2 \kappa^2 N^{2/\nu} + (c_0 + c_1 \kappa N^{1/\nu}) N^{-\omega} \quad (4)$$

In the linear approximation ($f_2 = c_1 = 0$), FSS predicts the intersection bewteen $B_4(\beta, N)$ and $B_4(\beta, N')$ denoted $(\beta^\star(N, N'), B_4^\star(N, N'))$

$$\begin{aligned} \beta^\star(N, N') &= \beta_c + \beta_c (c_0/f_1) L(N, N') \\ B_4^\star(N, N') &= B_4 + c_0 M(N, N') \qquad (5) \\ L(N, N') &= (N^{-\omega} - N'^{-\omega})/(N'^{1/\nu} - N^{1/\nu}) \\ M(N, N') &= (N^{-\omega-1/\nu} - N'^{-\omega-1/\nu})/(N'^{1/\nu} - N^{1/\nu}) \qquad (6) \end{aligned}$$

The nonlinear coefficient f_2 can be determined with a pencil and a ruler on a collapsed data graph.

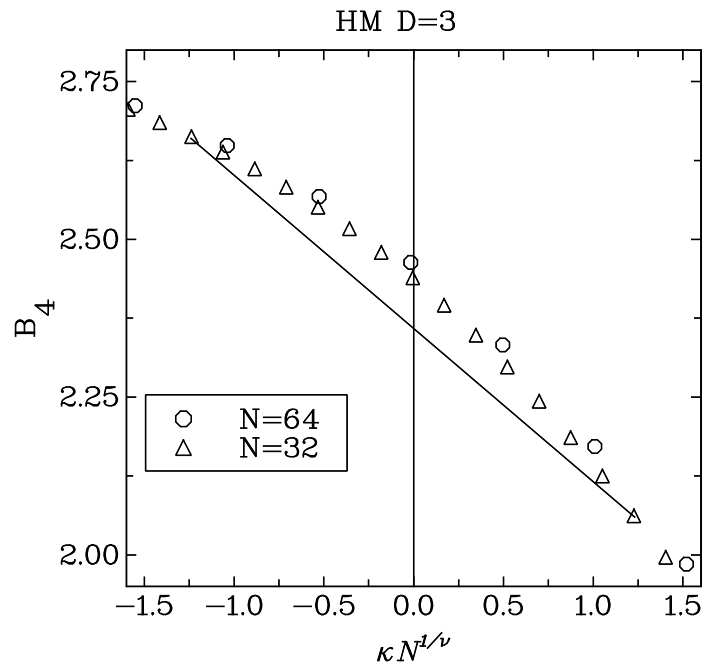

Figure 7: B_4 versus $\kappa N^{1/\nu}$, for $N = 32$ and 64 for the Ising hierarchical model. A "pencil and ruler" method yields $f_1 \simeq -0.24$ and $f_2 \simeq -0.045$ very good agreement with another independent estimate.

Fixed interval versus shrinking interval procedures

For each pair of linear sizes (N, N'), it is possible to determine the intersection $(\beta^\star(N, N'), B_4^\star(N, N'))$ of the corresponding linear fits. These empirical values are plotted versus the calculable values $L(N, N')$ and $M(N, N')$. However, if we increase the volume while keeping the κ interval constant, the accuracy of the extrapolations degrades rapidly (see Figs.).

We propose a shrinking interval procedure where the value of κ is restricted in such a way that

$$|\kappa| N^{1/\nu} < \epsilon f_1/f_2 \ . \tag{7}$$

as N increases. This procedure gives excellent results when N increases. This is illustrated for $\epsilon = 0.05$ in the following figures.

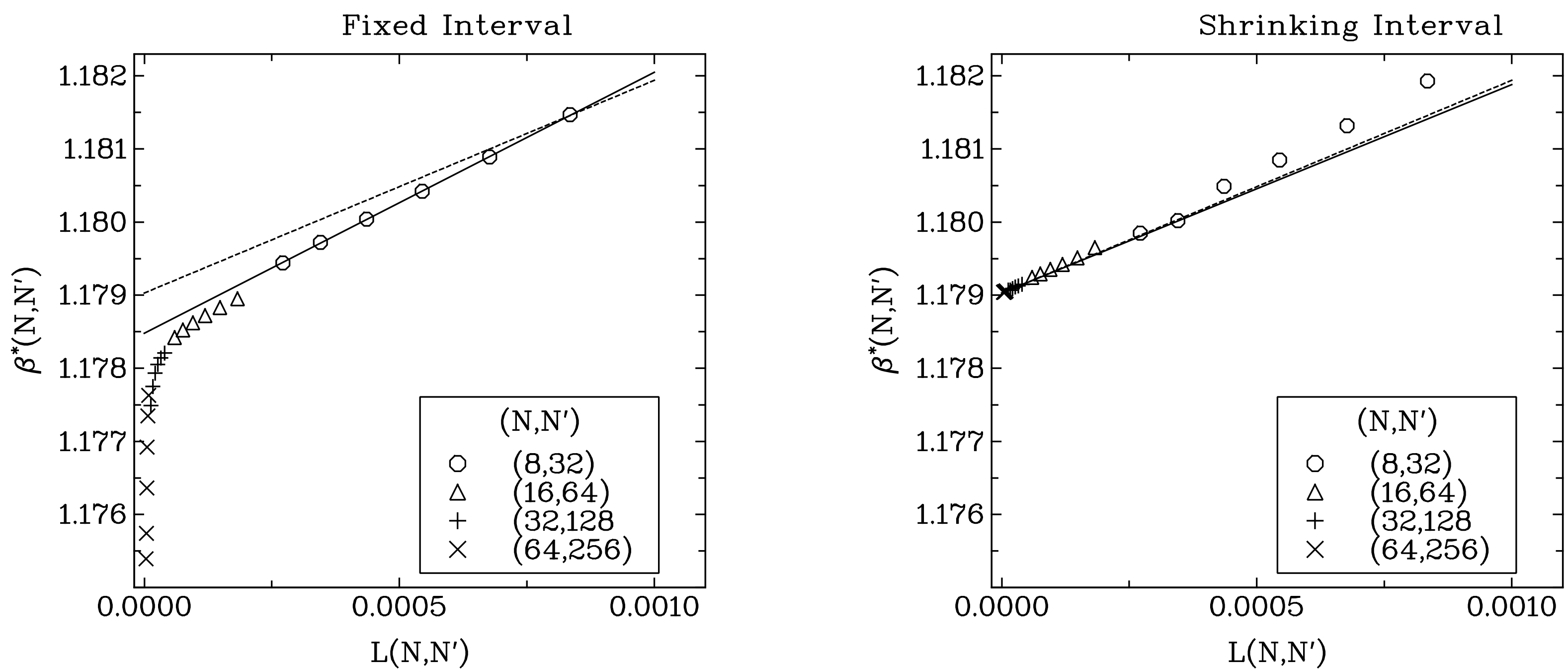

Figure 8: Empirical values of $\beta^\star(N, N')$ versus $L(N, N')$ for the two methods. The solid line is the best linear fit. The dash line is the exact behavior.

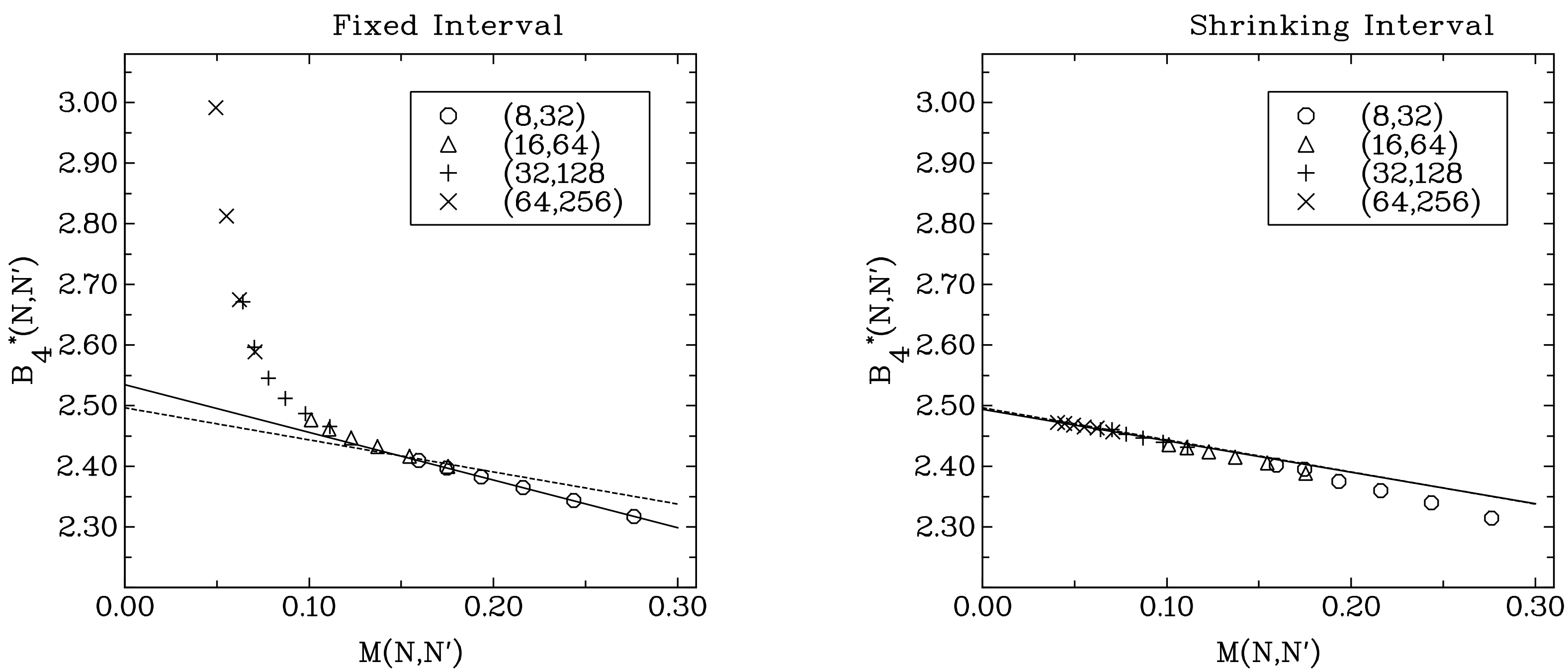

Figure 9: Empirical values of $B_4^\star(N, N')$ versus $M(N, N')$ for the two methods. The solid line is the best linear fit. The dash line is the exact behavior.