Department of Energy
Progress report Fall 2009 -Fall 2010

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Main Interests and Goals

Main interest: models of strong interactions primarily on a lattice

Applications: QCD and extensions beyond the standard model

Methods: improved perturbation theory and renormalization group methods

Grad. Students: Alan denBleyker, Daping Du, Yuzhi Liu and Haiyuan Zou

Computational facilities: Linux clusters here and at Fermilab

New computational possibilities explored: optical lattice realizations of lattice models

Graduate Students working with Y. Meurice

- **Daping Du** came in fall 2005. He has completed his Ph. D. work and all the other requirements to graduate and should finish writing and defend his dissertation this spring (2011). Works on the density of states and Fisher's zeros for $SU(2)$ lattice gauge theory (LGT). Fellowship from the graduate college last summer, T.A. in Fall 2010. He has received a URA fellowship to work at Fermilab on B physics in spring 2011. Travel to Fermilab in 2010 supported by our grant.

- **Alan Denbleyker** came in fall 2006. He works on MC simulations in $SU(2)$ gauge theories with and without adjoint terms and is planning to extend the existing codes for $SU(3)$. Ha has studied finite size scaling in $SU(2)$ LGT and the Ising model. He is the system manager for our cluster and repository. He is supported as a T.A. during the academic year and partially as a R.A. during summer. He has passed the qualifying exam and will take the comprehensive exam this spring 2011. RA support: 1.25 month in summer 2010.

- **Yuzhi Liu** came in fall 2006. He has passed the qualifying and comprehensive exams and the T.A. certification. He works on the comparison between discrete renormalization group methods that we have been using and continuous limits of these methods used by other authors and on complex RG flows in the hierarchical model. He has been supported partially as a T.A. and partially as a R.A. He has applied for a DOE graduate student fellowship in May 2010 but the proposal was not selected and will apply again in the coming weeks. R. A. support: 1.25 month in summer 2010.

- **Haiyuan Zou** came in fall 2008. He has passed the qualifying exam and the T. A. certification. He has been working on complex renormalization group flows in nonlinear sigma models and improved perturbative methods. He has been supported partially as a T.A. and partially as a R.A. (R. A. support: 4.25 months from March 2010 to February 2011, before TA certification).

Request: additional support to bring the students to Lattice 2011 conference held near San Francisco this year.

Recent Publications

Papers submitted or published since the submission of the proposal at the end of August 2009 and acknowledging DOE support. Except for A. Bazavov and A. Velytsky, all the authors are from the University of Iowa.

- Y. Meurice, *Dyson instability for 2D nonlinear O(N) sigma models*, Phys.Rev. **D80**, 054020 (2009).
- A. Denbleyker, Yuzhi Liu, Y. Meurice, and A. Velytsky, *Finite Size Scaling and Universality in SU(2) at Finite Temperature*, e-Print: arXiv:0911.1831 [hep-lat], PoS**LAT2009**197, 2009.
- A. Bazavov, A. Denbleyker, Daping Du, Y. Meurice, A. Velytsky, Haiyuan Zou, *Dyson's Instability in Lattice Gauge Theory* e-Print: arXiv:0910.5785 [hep-lat], PoS**LAT2009**218, 2009.
- A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, and Haiyuan Zou, *Fisher's zeros as boundary of renormalization group flows in complex coupling spaces*, Phys. Review Letters **104**, 251601, 2010.

- A. Bazavov, A. Denbleyker, Daping Du, Yuzhi Liu, Y. Meurice, and Haiyuan Zou, *Fisher's zeros as boundary of RG flows in complex coupling space* arXiv:1011.1675, submitted to the POS for Lattice 2010.

Preprints at a first draft stage

- Y. Liu and Y. Meurice, *Complex RG Flows for Dysons Hierarchical Model*.
- Y. Meurice and H. Zou, *Complex RG Flows for 2D Nonlinear* $O(N)$ *Sigma Models*.
- A. Denbleyker, D. Du, Y. Meurice, and A. Velytsky, *Fishers Zeros of* $SU(2)$ *Lattice Gauge Theory*.
- A. Bazavov, Daping Du, and Y. Meurice, *Density of States and Fisher's zeros in* $U(1)$ *pure gauge theory.*
- Y. Liu and Y. Meurice, *About the continuum limit of discrete RG transformations.*

Presentations since September 2009:

Presentations by Y. Meurice at:

- Quantum gauge theories and ultracold atoms, Sant Benet, Sept. 2009.
- Nuclear Theory get together, Argonne, October 2009.
- New applications of the RG method, U. Washington Seattle, Feb. 2010.
- Washington University, St Louis, March 2010.
- Aspen Center for Physics, June 2010.
- Lattice 2010, Villasimius, June 2010.
- Univ. of Utrecht , August 2010.
- 5th ERG Conference, Corfu, September 2010.
- KITP Conference: Frontiers of Ultracold Atoms and Molecules, Oct. 2010 .
- UCLA, October 2010.
- UC Riverside, October 2010.

Talks given by students

- Haiyuan Zou, "Volume Effect of Fisher zeros in the Nonlinear sigma model", Meeting of the Prairie Section of the APS, November 2009; Iowa City.
- Yuzhi Liu, "Finite Size Scaling and Universality in SU(2) Lattice Gauge Theory at Finite Temperature", Meeting of the Prairie Section of the APS, November 2009; Iowa City.
- Haiyuan Zou, "Fisher zeros, singularities of the gap equation and zeros of the beta function for nonlinear O(N) sigma models at finite volume", INT-10-45W, February 2010.
- Yuzhi Liu, "Numerical instabilities associated with block spinning non-integer numbers of sites", INT-10-45W, February 2010.

Conference organization and related activities

I co-organized two workshops:

- *New Applications of the Renormalization Group Method*, INT workshop, Feb. 22-26, 2010, with M. Birse, and S.-W.Tsai ; 35 participants, including 2 U. Iowa students.
- *Critical Behavior of Lattice Models*, Aspen Workshop, May 24 -June 11 2010, with G. Baym, U. Schollwoeck and S.-W. Tsai; 43 participants.

One proposal for a five weeks program on criticality in lattice models at the Kavli Institute for TheoreticalPhysics in China in July-August 2012 has been selected. The International Coordinating Board will be Lu-ming Duan (U. Michigan), Yannick Meurice (U. Iowa), Shan-Wen Tsai (UC Riverside), Xiao-gang Wen (MIT) and Zhenghan Wang (MicrosoftQ).

I am a guest editor for the a theme issue of the Ph. Trans. A of the Royal Society on recent applications of the RG method (10 contributions, scheduled to appear in spring 2011).

Upgrade of computer facilities (Alan denBleyker)

2006: new cluster with 8 single CPU nodes with 3.2 GHz Pentium 4 processors and Gigabyte motherboards with a build-in fast ethernet card (still in operation but slow).

June 2009: 3 nodes with 4GB of Ram, 2.33Ghz Core2 Quad processors, sata hard drives.

April 2010: 7 more nodes using essentially the same components. The combined cost was $3337 or $334 per computer of which each has 4 cores. The upgrades have been paid by the Department of Physics and Astronomy using overhead return.

Desktops: Pentium 4 single core 3.2Ghz with 2GB of memory from 2005; Pentium 4 dual core 3.0Ghz with 2GB of memory from 2006; Pentium 4 dual core 3.4Ghz with 2GB of memory from 2007 (after a motherboard replacement and ram replacement under warranty by Dell, the computer continued to be plagued with random shutdowns and bluescreens). We would like to replace 2 of these computers with newer models.

Summary of Recent Progress

- There has been a renewed interest in the lattice community for nontrivial infrared fixed points in asymptotically free gauge theories. In order to decide if candidate models beyond the standard model confine, we proposed new methods to extend the RG transformation to complex coupling spaces and found that the Fisher's zeros are located at the boundary of the complex basin of attraction of infra-red fixed points. We supported this picture with numerical calculations and discussed the implications for proofs of confinement and searches for nontrivial infra-red fixed points in models beyond the standard model. The results appeared in *Phys. Review Letters* **104** in June 2010 with all U. Iowa authors. The details will appear in 3 other papers which are in an advanced stage of preparation.
- Recently, Daping Du started to work on the branching ratio for the decay $B_s \to \mu^+\mu^-$ (which is sensitive possible new physics beyond the Standard Model) with the Fermilab lattice group. Daping Du has been awarded a URA fellowship to stay at Fermilab in spring 2011 and work on this project.

- In order to understand the large order behavior of perturbative series in QCD, we have started to work on the $SU(2)$ coefficients of the average plaquette in lattice gauge theory with Francesco di Renzo on our cluster. A first run with low statistics has provided the first 10 coefficients consistent with the low orders exact results and with ratio of successive coefficients near 2 as expected. A proposal of class C at Fermilab to continue this work on the Fermilab clusters has been approved. We plan to work on the question of large field configurations in stochastic perturbation theory next year.
- The idea of using optical lattices to do lattice gauge theory calculations that I presented in a poster a recent KITP conference "Frontiers of Ultracold Atoms and Molecules" (http://online.itp.ucsb.edu/online/boptilatt-c10/meurice/) has received interest from the cold atom community and could possibly lead to funding from Atomic and Molecular physics.

Recent Work on complex RG flows

The phenomenological importance of controlling the height of the β function (for instance in "walking tchnicolor" models) motivated us to study extensions of renormalization group (RG) flows in the complex coupling plane. A general feature that we observed is that the Fisher's zeros - the zeros of the partition function in the complex coupling plane - apparently act as "gates" for the RG flows ending at the strongly coupled fixed point. For confining theories, the gate stays open as the volume increases and flows starting in a complex neighborhood of the UV fixed point can reach the IR fixed point where confinement and the existence of a mass gap are clearly present. In general, losing conformality corresponds to the generation of a mass gap and the presence of confinement and complex fixed points not on the real axis. We argued that such fixed points are related to the absence of Fisher's zeros on the real axis. We studied this scenario with model calculations for $2D$ $O(N)$ non-linear sigma models in the large-N limit and the Ising hierarchical model. We plan to pursue this work for abelian and non-abelian theories theories with fermions.

Complex RG flows in the hierarchical model (with Yuzhi Liu)

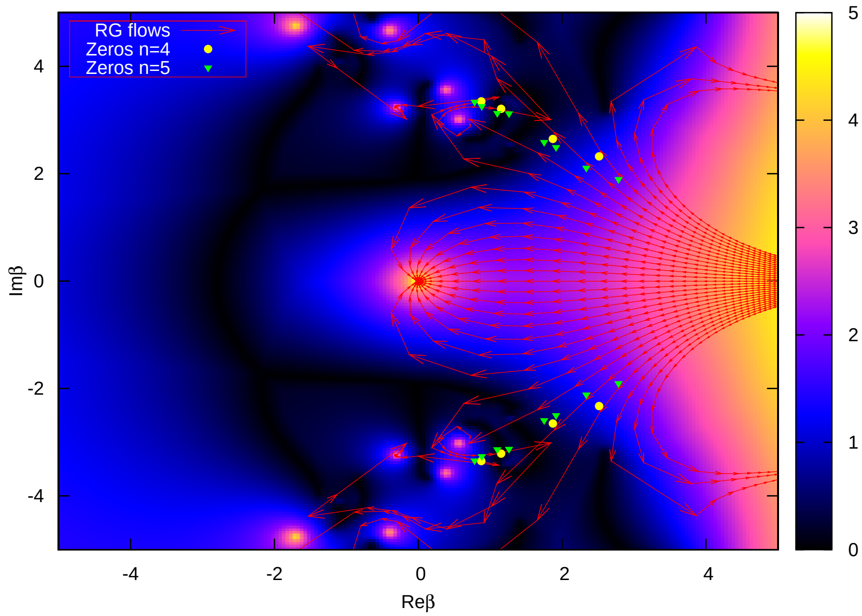

Figure 1: RG flows obtained by the two lattice matching methods for Hierarchical Model. The flows remain within the boundary formed by the Fisher's zeros.

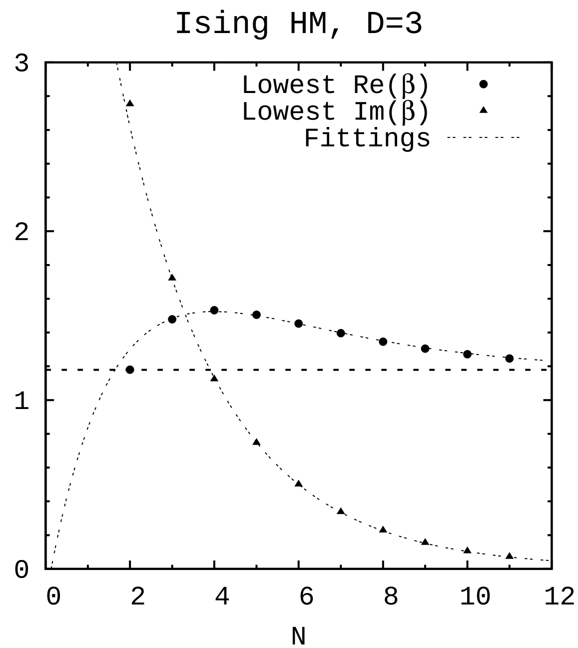

Figure 2: Volume dependence of the real and imaginary part of the zero with respect to the infinite volume critical point. The dotted lines come from finite size scaling theory.

Complex RG flows in $O(N)$ models (with Haiyuan Zou)

For the $O(N)$ models, we constructed the Riemann sheet structure and singular points of the finite lattice size L mappings between the mass gap and the 't Hooft coupling.

We argued that the Fisher's zeros appear on "strings" ending approximately at the singular points mentioned above.

We showed that for the spherical model at finite N and L, the density of states is stripwise polynomial in the complex energy plane.

We compared finite volume complex flows obtained from the rescaling of the ultraviolet cutoff in the gap equation and from the two lattice matching. In both cases, the flows are channelled through the singular points and end at the strong coupling fixed points, however strong scheme dependence appear on the ultraviolet side.

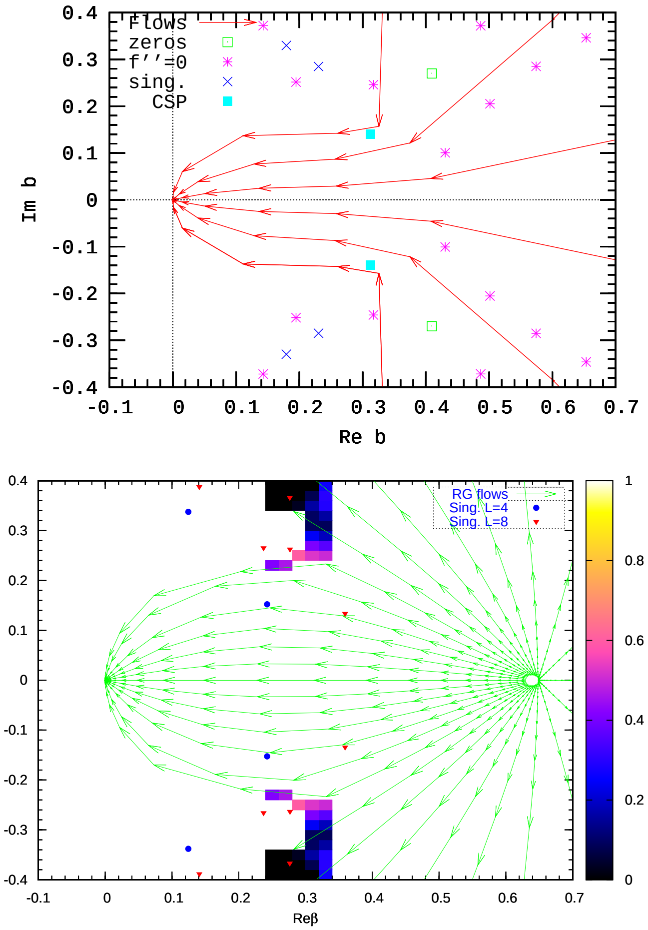

Figure 3: a) RG flows by rescaling (up) and two lattice matching (down) at finite volume.

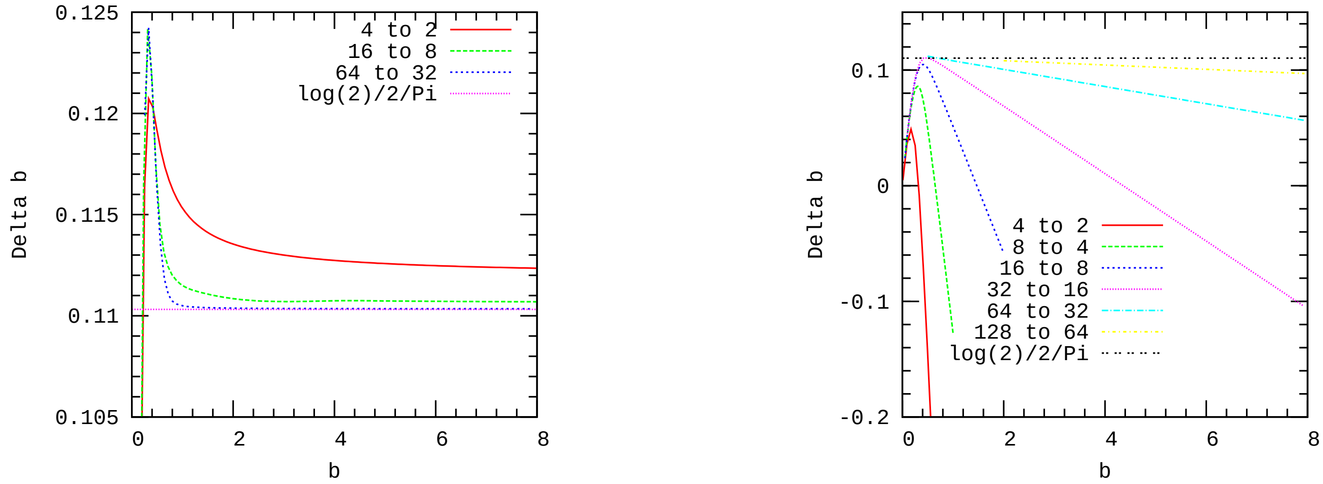

Figure 4: Nonperturbative β function: Δb versus b from rescaling (left) and 2-lattice matching (right).

Fisher' s Zeros in $U(1)$ lattice gauge theory (with D. Du and A. Bazavov)

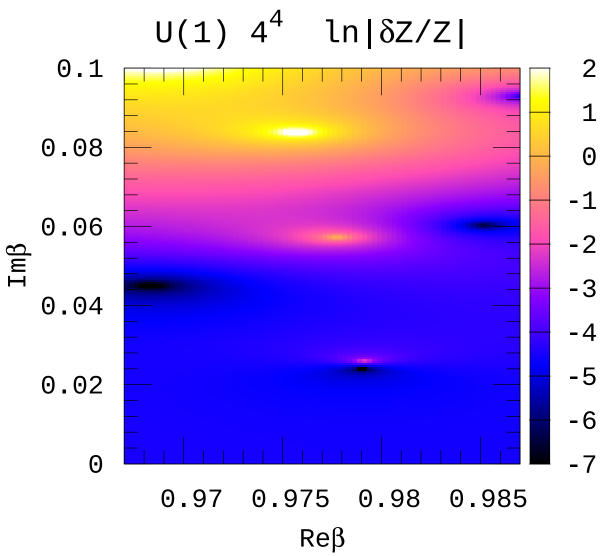

Figure 5: $|\delta Z/Z|$ for $U(1)$ on 4^4 lattice. In the $U(1)$ case, multicanonical methods were used and naive histogram reweighting works well. The numerical error δZ can be estimated from $(n_i(S) - < n(S) >)$, where i is an index for independent runs.

Fisher' s Zeros for $SU(2)$ lattice gauge theory (with D. Du and A. denBleyker)

For $SU(2)$, the imaginary part of Fisher's zeros are too large to use simple reweighting methods. We used the Ferrenberg-Swendsen method and checked its convergence.

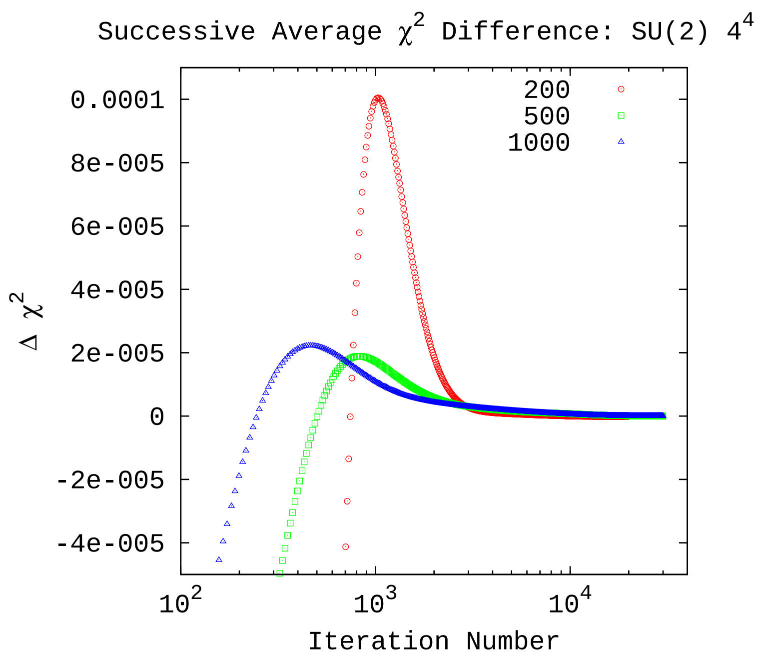

Figure 6: As an example, the successive difference of the average χ^2 of the average plaquettes is monitored during the iterations which are on a SU(2) gauge field on a 4^4 lattice. The red, green and blue are corresponding to the bin number 200,500 and 1000, which indicates that convergence is independent of histogram settings.

By using Chebyshev interpolation for $f(s)$ and monitoring the numerical stability of the integrals with the residue theorem, it is possible to obtain reasonably stable results. Unlike the $U(1)$ case, the imaginary part of the lowest zeros does not decrease as the volume increases, but their linear density increases at a rate compatible with L^{-4}. The effect of an adjoint term (+0.5) is that the lowest zero goes down by about 40 percent.

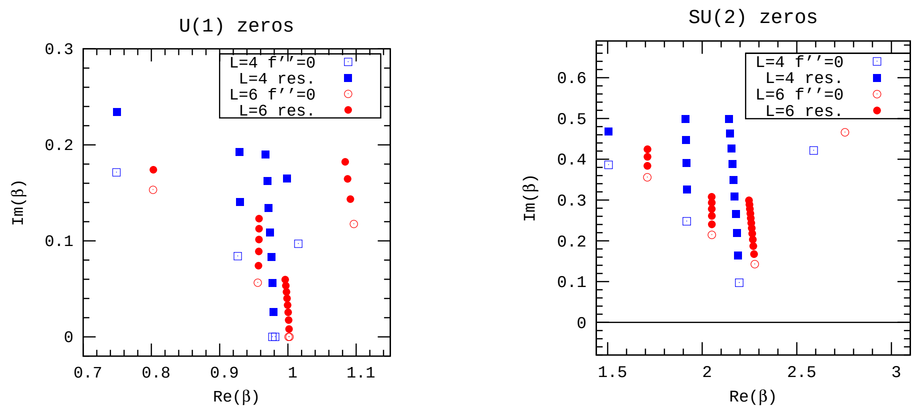

Figure 7: Images of the zeros of $f''(s)$ in the β plane (open symbols) and Fisher's zeros (filled symbols) for $U(1)$ and $SU(2)$ on 4^4 (squares) and 6^4 (circles) lattices.

Numerical instabilities for non-integer blocking (with Y. Liu)

We proposed an extension of the recursion formula of Dyson's hierarchical model where the number of sites blocked becomes an arbitrary number bD instead of 2 in the original formula.

We showed that when bD is an integer, the polynomial approximations developed for $bD = 2$ remain valid. The value of the critical exponent ν depends slightly on b and changes by 0.0012 between $bD = 2$ and $bD = 8$. When bD is not integer, the polynomial approximation breaks down at a degree l_{max} which decreases with bD. We explain this instability by considering $bD = 2 + \zeta$ and expanding at first order in ζ.

We also found a remarkable relation between ν and ω falling on a curve found by Litim.

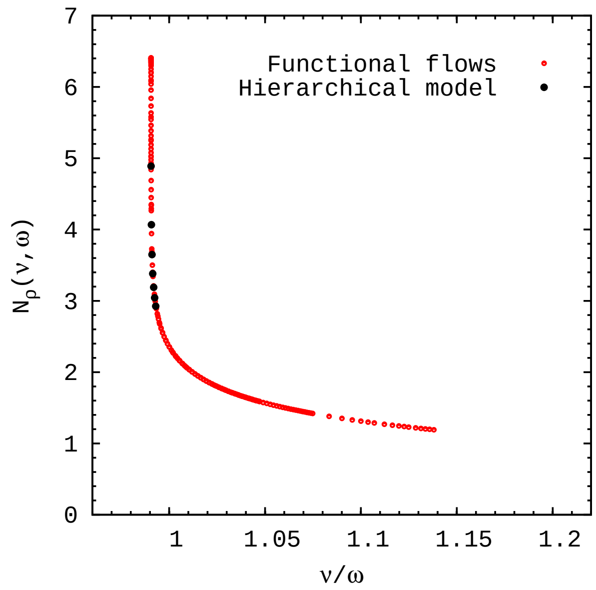

Figure 8: Relation between ν and ω.

BR($B_s \to \mu^+\mu^-$) from the Semileptonic Decays (Daping Du with the Fermilab/MILC Coll.)

In recent years, increasing interest has been placed on the rare decay $B_s^0 \to \mu^+\mu^-$ where the Penguin and Box topologies dominate. Due to the GIM mechanism, the BR($B_s \to \mu^+\mu^-$) is very small $(3.6 \pm 0.4) \times 10^{-9}$ and an observed discrepancy would open a window on possible physics beyond the standard model. The upper bound from the CDF and D0 collaboration is around 4.3×10^{-8}(CDF/D0 2009). At LHCb, the branching ratio will be obtained by using comparison with other normalization channels like $B_u^+ \to J/\psi K^+$ or $B_d^0 \to K^+\pi^-$ in the following manner:

$$BR(B_s^0 \to \mu^+\mu^-) = BR(B_q \to X)\frac{f_q}{f_s}\frac{\epsilon_X}{\epsilon_{\mu\mu}}\frac{N_{\mu\mu}}{N_X}$$

where the efficiencies ϵ and count numbers N come from experimental measurements. Significant uncertainty comes from the ratio of fragmentation functions f_q/f_s for which a precise estimate is yet to be seen.

Daping Du has started to work with Andreas Kronfeld and the Fermilab Lattice/MILC collaboration on a lattice calculation of the the fragmentation function ratio f_q/f_s which can be calculated by extracting the ratio $BR(\bar{B}_s^0 \to D_s^+\pi^-)/BR(\bar{B}_d^0 \to D^+K^-)$ which will reduce, by factorization, to the form factor ratio of the semileptonic decays $F_0^{B_s\to D_s}(m_K^2)/F_0^{B\to D}(m_\pi^2)$. As a result, the fragmentation ratio can be expressed explicitly by (Fleischer et al.)

$$\frac{f_d}{f_s} = 12.88\frac{\tau_{B_s}}{\tau_{B_d}}\frac{\epsilon_{D_s\pi}}{\epsilon_{D_dK}}\frac{F_0^{(s)}(m_\pi^2)}{F_0^{(d)}(m_K^2)}\frac{a_1(D\pi)}{a_1(DK)}/\frac{N_{D_s\pi}}{N_{D_d\pi}}$$

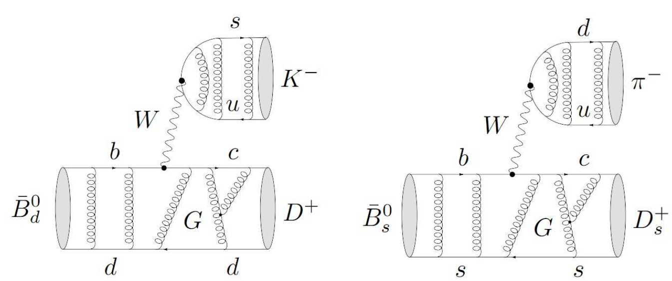

Figure 9: Fig. and formula from Fleisher et al. Phys.Rev.D82:034038,2010

Gauge configurations with 2+1 flavors generated by the MILC collaboration are being used to extract the semileptonic form factors by realizing the decay processes on a lattice. The two heavy quarks b and c are described with the Fermilab action while the light spectator quarks s and d are described by AsqTad improved staggered fermions. The fitting of the two-point and three-point corelators is based on the heavy quark symmetry. In order to determine the fragmentation ratio f_s/f_d, the form factors are then extrapolated through the z-parametrization to the masses of the light mesons $M_{K(\pi)}$.

Project Status: The project started in August 2010 and is still in progress. Half of the data, which is for the $B \to Dl\nu$ decays, are made ready by the Fermilab/MILC Collaboration and a few ensembles for the $B_s \to D_s l\nu$ have been generated. The project is in the phase of data generating/analyzing. Preliminary results are expected in February or March.

Strategies for Optical Lattices

The possibility of doing lattice gauge theory simulations using optical lattices can be seen more easily in the Hamiltonian formulation (in 2 or 3 space dimensions). In the temporal gauge, the unitary matrices in the time direction are gauged transformed to the identity and the quantum hamiltonian has the form

$$H = \frac{g^2}{2} \sum_{space\ links} E^a E^a - \frac{2N}{g^2} \sum_{space\ plaq.} (1 - (1/N) ReTr(U_p))$$

with $E^{ia}(\mathbf{x}, t) \propto tr(\dot{U}^{\dagger}_{(\mathbf{x},t),\mathbf{e}_i} T^a U_{(\mathbf{x},t),\mathbf{e}_i})$ the color electric fields. They can be seen as the generators of the local gauge transformations. They obey local commutation relations similar to the Lie algebra and the $U_{(\mathbf{x},t),\mathbf{e}_i}$ transform like the adjoint representation under commutation with $E^{ia}(\mathbf{x}, t)$. The generic form of the gauge boson interactions with fermions (quark-gluon interaction in QCD) is

$$\sum \bar{\psi}^a_{(\mathbf{x},t)} \gamma^i U^{ab}_{(\mathbf{x},t),\mathbf{e}_i} \psi^b_{(\mathbf{x}+\mathbf{e}_i,t)}$$

As explained above, it is essential to have dynamical $U^{ab}_{(\mathbf{x},t),\mathbf{e}_i}$ in order to obtain the main physical features. This also appears to be the most challenging part of the program. I see two possible types of strategies:

- **Strategy I: quantum gauge fields and fermions**
  Engineer quantum link variables having an hamiltonian with plaquette interactions . This possibility seem to require an underlying local gauge symmetry. Correlation functions of gauge invariant products of fermions could be measured by introducing local source parameters coupled linearly to the gauge invariant products of fermion fields and taking "functional variations" as in quantum field theory.
- **Strategy II: MC gauge variables and quantum fermions**
  Alternatively, one could use numerical link variables of MC simulations and replace the fermion determinants and propagators calculations in a fixed configuration for the link, by measurements of fermion correlations on the optical lattice. This possibility requires the ability to manipulate locally the hopping parameters and to have fast enough communication between the classical computer and the optical lattice.

Challenges

This is a list of problems that need to be solved in order to implement the above strategies.

- **Relativistic fermions with global color**
  Using three of the hyperfine levels F=1/2 and 3/2 of ^{6}Li Fermi gas near a Feshbach resonance, one can create a quantum degenerate three-state Fermi gas with approximate $SU(3)$ symmetry On a honeycomb lattice, a single flavor Dirac theory with global $SU(3)$ symmetry could be obtained.
  Interesting ways of coupling Dirac fermions to periodic or staggered gauge potentials by combining two types of square lattices have also been proposed.
- **Dynamical link variables**
  An idea that would come naturally to a particle physicist who was a graduate student in the technicolor era is to build the link variable $U_{\mathbf{x},i}{}^{ab}$ as a "condensate" of the site variables $\phi^a_{\mathbf{x}}$ at the ends of the link

$$U^{ab}_{\mathbf{x},\mathbf{e}_i} = {\phi^{\star}}^a_{\mathbf{x}} \phi^b_{\mathbf{x}+\mathbf{e}_i} \ .$$

Directional or summed "hypercolor" indices could be added.

- **Local manipulation of hopping parameters**
  Global non-abelian Berry phases can be obtained from adiabatic transformations in degenerate quantum mechanical systems. Such phases can be obtained from "dark states" in a tripod system. Global $SU(N)$ potentials can also be created using N internal states of atoms and laser assisted state sensitive tunnelling I am not aware of attempts to make these constructions local. However, locally rotating deformations of optical lattice have been studied recently.
- **Local symmetry?**
  The principle of local gauge symmetry has played a central role in the development of the standard model of all known non-gravitational interactions. I believe it is also central for the present project. Local symmetry emerges in trapped alkali with hyperfine states and the gauge field is the superfluid velocity.

- **Plaquette interactions**
  Maybe the most challenging part of Strategy I is to create plaquette interactions. A possibility suggested by Cheng Chin is to use two lattices one having molecules that can hop and induce the desired interactions on the other lattice.