Progress Report of Yannick Meurice

Our main goal is to develop new field theoretical methods which can be used in situations where perturbative methods fail. This involves conventional perturbative methods combined with a large field cutoff, the $1/N$ expansion, semi-analytical calculations of renormalization group flows and Monte-Carlo simulations in lattice gauge theory. Our objective is to bring higher standards of accuracy in quantum field theory and ultimately to be able to make predictions that can be compared with experiments which emphasize precision ($g-2$, hadronic width of the Z, etc...). In 2003, we built a 16 nodes cluster that has allowed us to create large numbers of gauge configurations on lattices with up to 16^4 sites and to work on scalar field theory in various dimensions. Operating this cluster is a full time job that has been carried succesfully by our graduate student Li Li. In our budget, we request 9 months of graduate student support and 2,700 dollars to upgrade the communication to Gigabit ethernet cards. Our recent work on gauge theories is reported in section 1 and on scalar models in section 2.

1. Work on Gauge Theories

a. Gluodynamics at negative g^2. We considered Wilson's $SU(N)$ lattice gauge theory (without fermions) at negative values of $\beta = 2N/g^2$ and for N=2 or 3. We showed that in the limit $\beta \to -\infty$, the path integral is dominated by configurations where links variables are set to a nontrivial element of the center on selected non intersecting lines. For $N = 2$, these configurations can be characterized by a unique gauge invariant set of variables, while for $N = 3$ a multiplicity growing with the volume as the number of configurations of an Ising model is observed. In general, there is a discontinuity in the average plaquette when g^2 changes its sign which prevents us from having a convergent series in g^2 for this quantity. For $N = 2$, a change of variables relates the gauge invariant observables at positive and negative values of β. For $N = 3$, we derived an identity relating the observables at β with those at β rotated by $\pm 2\pi/3$ in the complex plane and showed numerical evidence for a Ising like first order phase transition near $\beta = -22$ illustrated in Fig. 1.

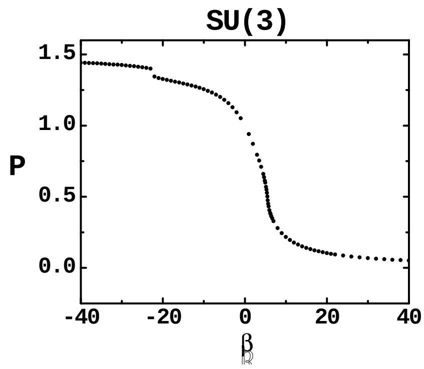

Figure 1: The average action density $P(\beta)$ for $SU(3)$

Publication: L. Li and Y. Meurice, Lattice gluodynamics at negative g^2, hep-lat/0410029, 2004, Phys. Rev. D (submitted).

b. A possible third order phase transition in 4D gluodynamics. We revisited the question of the convergence of lattice perturbation theory for a pure $SU(3)$ lattice gauge theory in 4 dimensions. Using the most recent calculation of the weak coupling expansion of the plaquette average, we showed that the extrapolated ratio and the extrapolated slope suggest a nonanalytical power behavior at $\beta = 6/g^2 \simeq 5.7$ with an exponent $\gamma \simeq -1.1$. We found indications for a possible singularity in the third derivative of the free energy on 8^4 and 10^4 lattices (see Fig 2). As the lattice size increases, the statistical errors become large and a significantly larger number of independent configurations is needed in order to draw definite conclusions.

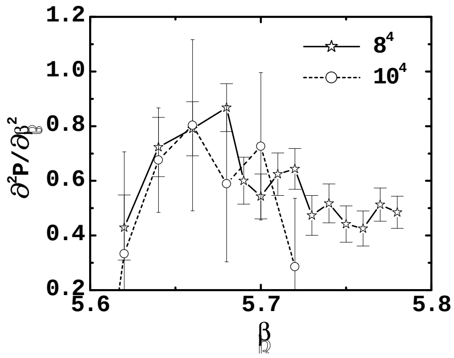

Figure 2: Second derivative of P versus β.

Publication in progress: L. Li and Y. Meurice, A possible third order phase transition in 4D gluodynamics, draft available at http://www-hep.physics.uiowa.edu/∼meurice/drafts/lpt.ps

c. Local versus nonlocal field cuts. For scalar fields, the configurations can be ranked according to the largest absolute value of the field or according to the average over the sites of an even power of the field. The largest this power is, the more emphasis is put on the configurations with the largest field values. We expect correlations among these quantities. This is illustrated in Fig. 3. Discarding the large field configurations changes the large order behavior of perturbative series. Out of the 10,000 configurations only 42 have values of $|\phi|$ larger than 3. Neglecting these configurations affect the the order λ correction to E_0 ($\langle 0|\phi^4|0\rangle = 3/4$ without a field cut) by 1 percent. The same 42 account for about 90 percent of the sixth coefficient! We have attempted to follow the same procedure for gauge models using the Landau gauge where we believe that $1 - (1/N)ReTrU_{link}$ should play a role analogous to ϕ^2 in scalar models. We found correlations between the lattice average of this quantity and the average action. However, we found no correlations between the average and the maximum value. This is probably due to the imperfect way the Landau gauge condidtion is implemented numerically (in the large public quenched configuarations that we have used). We are planning to study the maximum value as a function of the computer time while the Landau gauge algorithm is being implemented.

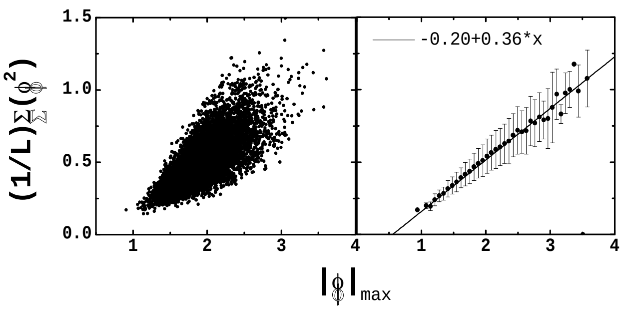

Figure 3: Largest absolute value of the field versus average over all the sites of ϕ^2, in $D = 1$ (harmonic oscillator), for 10,000 configurations; sample correlation: 0.73.

Publication: L. Li and Y. Meurice, Effects of large field cutoffs in scalar and gauge models, hep-lat/0409096 (to appear in the Lattice 2004 proceedings).

d. A proposal for a "perfect" field cut in Lattice gauge perturbation theory. We considered the effects of a field cutoff on the weak coupling series of a one plaquette $SU(2)$ lattice gauge theory. It possible to pick a the (perfect) field cutoff in such a way that the series *converges* toward the *correct* answer. We are considering the implementation of the method with a Langevin equation and its extension for four dimensional lattice gauge theory.

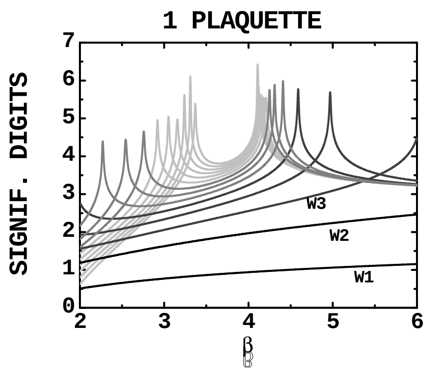

Figure 4: Number of correct significant digits as a function of β as the order increases, the curves gets less dark (more grey). The cut was chosen to be perfect at $\beta = 4$.

Publication in progress: L. Li and Y. Meurice, A perfect field cut in Lattice gauge perturbation theory, draft at http://www-hep.physics.uiowa.edu/∼meurice/drafts/perfect.ps

2. Work on scalar models

a. Small numerators cancelling small denominators of the HT scaling variables. We found a method to express the susceptibility and higher derivatives of the free energy in terms of the scaling variables (Wegner's nonlinear scaling fields) associated with the high-temperature (HT) fixed point of Dyson hierarchical model in arbitrary dimensions. We found a closed form solution of the linearized problem. We checked that up to order 7 in the HT expansion, all the poles ("small denominators") that would naively appear in some positive dimension are canceled by zeroes ("small numerators"). The requirement of continuity in the dimension can be used to lift ambiguities which appear in calculations at fixed dimension. We showed that the existence of a HT phase in the infinite volume limit for a continuous set of values of the dimension, requires that this mechanism works to all orders. On the other hand, most poles at negative values of the dimensional parameter (where the free energy density is not well-defined, but RG flows can be studied) persist and reflect the fact that for special negative values of the dimension, finite-size corrections become leading terms. We showed that the inverse problem is also free of small denominator problems and that the initial values of the scaling variables can be expressed in terms of the infinite volume limit of the susceptibility and higher derivatives of the free energy.

Publication: Y. Meurice, Small numerators cancelling small denominators of the HT scaling variables: a systematic explanation in arbitrary dimensions, Phys.Rev. E69 (2004) 056108.

b. Universality in nontrivial continuum limit. We studied numerically the continuum limit corresponding to the non-trivial fixed point of Dyson's hierarchical model. We found that the critical amplitudes as input parameters. We determined numerically the leading and subleading critical amplitudes of the zero-momentum connected $2l$-point functions in the symmetric phase up to the 20-point function for randomly chosen local measures. Using these amplitudes, we constructed quantities which are expected to be universal in the limit where very small log-periodic corrections are neglected: the $U^{(2l)\star}$ (proportional to the connected $2l$-point functions) and the r_{2l} (proportional to one-particle irreducible(1PI)). We showed that these quantities are independent of the the local measure with at least 5 significant digits. We provided clear evidence for the asymptotic behavior $U^{(2l)\star} \propto (2l)!$ and reasonable evidence for $r_{2l} \propto (2l)!$. These results signal a finite radius of convergence for the generating functions. We provided numerical evidence for a linear growth for universal ratios of subleading amplitudes. We compared our r_{2l} with existing estimates for other models.

Publication: Y. Meurice and B. Oktay, Universality in nontrivial continuum limits: a model calculation, Phys.Rev.D69 (2004) 125016.

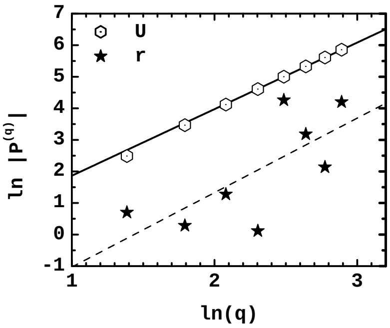

Figure 5: Values of $\ln|r_{l+2}/r_l|$ versus $\ln 2l$. The stars correspond to ratios of r_{2l}. The hexagons correspond to the same quantity but with r_{2l} replaced by $U^{(2l)*}$. The lines are linear fits, the slope of the solid line is 2.11 while the slope of the dashed line is 2.36.

c. About the Borel summability of the $1/N$-expansion of the hierarchical $O(N)$ models. We calculated numerically the critical exponents and critical temperature of the $3D$ $O(N)$ hierachical models at large but finite N. We found the rate at which the degree of polynomial truncations used to get a fixed accuracy, increases with N. From these numerical values, we extracted the coeffients of the $1/N$ expansion up to order 6 for the temperature and order 4 for the exponents. The method of calculation of the coefficients using numerical values, was developed with the Sterling series for which we were able the first 7 coefficients accurately. Using the Padé-Borel method, we reproduce accuratrely the critical quantities at low N. Our results are consistent with the hypothesis that the $1/N$ expansions considered are asymptotic but Borel summable.

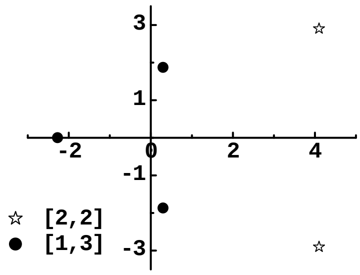

Figure 6: The pole of Padé approximants

Publication in progress: L. Li and Y. Meurice, About the Borel summability of the $1/N$-expansion of the hierarchical $O(N)$ models, preprint in pogress, a draft is available at http://www-hep.physics.uiowa.edu/∼meurice/drafts/Ndraft.ps

d. Optimization Methods for Converging Perturbative Series with a Field Cutoff. In $\lambda\phi^4$ problems, a large field cutoff ϕ_{max} makes perturbative series *converge* toward values exponentially close to the exact values. It is possible to make optimal choices of ϕ_{max}. For perturbative series terminated at even order, it is possible to adjust ϕ_{max} in order to obtain the exact result. For perturbative series terminated at odd order, the error can only be minimized. We found weak and strong coupling methods to determine ϕ_{max}. For a simple integral, we compared our methods with the linear δ-expansion (LDE) (combined with the principle of minimal sensitivity) which provides an upper envelope of for the accuracy curves of various Padé and Padé-Borel approximants. Our optimization method performs better than the LDE at strong and intermediate coupling, but not at weak coupling where it appears less robust and subject to further improvements.

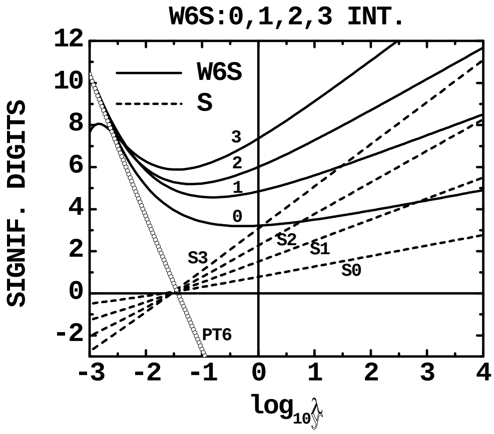

Figure 7: Significant digits obtained with the optimal cut $\phi_{max}(\lambda)$ (corresponding to a truncated expansion at order 6 in the weak coupling) estimated using the strong coupling expansion at orders 0, 1, 2 and 3 (solid lines), compared to significant digits using only the strong coupling expansion of the integral at the same orders in the strong coupling (dashed lines) and regular perturbation theory at order 6 (PT6).

Publication: B. Kessler, L. Li and Y. Meurice, New Optimization Methods for Converging Perturbative Series with a Field Cutoff, Phys.Rev. D69 (2004) 045014.

e. Monte Carlo calculation of perturbative coefficents with a large field cutoff. We calculated perturbative coefficients for lattice scalar field theory using the MC method in dimensions 1, 2 and 3, where the large field contributions are cutoff. This produces converging (instead of asymptotic) perturbative series. We discussed the statistical errors and the lattice effects and showed that accurate calculations are possible even in a crossover region where no approximation works. We show that the field cutoff is also a UV regulator.

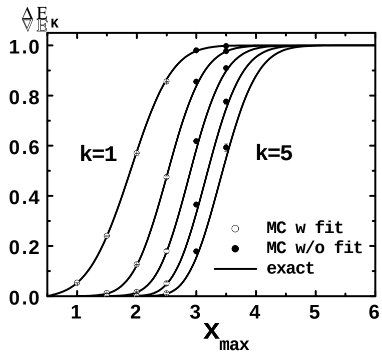

Figure 8: The comparison of Monte Carlo result (circles) and accurate numerical result (continuous line), all the values has been divided by their infinite cut limits

Publication in progress: L. Li and Y. Meurice, Monte Carlo calculation of perturbative coefficents, draft available http://www-hep.physics.uiowa.edu/ meurice/drafts/mc.ps