Here is a collection of problems and readings to work on through spring break. The problems are due 21 March.

1. Self Study: In Jackson, finish reading Chapter 7 to become familiar with different types of dispersion that you might encounter in your work/research.

2. Self study: Begin reading chapter 8 on wave guides. I will not cover this material in class. Do problems 8.2 and 8.5.


4. The Bianchi Identities: From last semester in HW3 recall that the Riemann Curvature tensor, $R_{ijkl}^\ m$ is defined through the covariant derivative operator as

$$\nabla_i \nabla_j K_l - \nabla_j \nabla_i K_l = R_{ijkl}^\ m K_m, \tag{1}$$

where $K_m$ is an arbitrary rank one covariant tensor. It was then straightforward to show that

$$(\nabla_a \nabla_b - \nabla_b \nabla_a)K_{cd} = R_{abc}^\ m K_{md} + R_{abd}^\ m K_{cm}.$$  

Now since the covariant derivative operator is a derivation it must satisfy the associativity condition which requires that:

$$([\nabla_a, [\nabla_b, \nabla_c]] + [\nabla_c, [\nabla_a, \nabla_b]] + [\nabla_b, [\nabla_c, \nabla_a]]) N_d = 0,$$

regardless of the vector $N_d$. By isolating the terms proportional to $N_d$ and $\nabla_p N_d$ show that

$$R_{ijkl}^\ m + R_{ijl}^\ m + R_{jli}^\ m = 0 \quad \text{First Bianchi Identity} \tag{2}$$

and that

$$\nabla_k R_{ijkl}^\ m + \nabla_j R_{kild}^\ m + \nabla_i R_{jkl}^\ m = 0. \quad \text{Second Bianchi Identity} \tag{3}$$

5. Einstein’s Equation: From the second Bianchi identity, the definition of the Ricci tensor $R_{ab}$ and the scalar curvature $R$, show that

$$\nabla_a G^{ab} \equiv \nabla_a \left( R^{ab} - \frac{1}{2} g^{ab} R \right) = 0.$$ 

The term in the parenthesis, $G^{ab}$ is called the Einstein tensor. Since this is naturally divergent free (i.e. $\nabla_a G^{ab} = 0$) it suggests a conservation law. Einstein therefore equates the $G^{ab}$ tensor to the energy-momentum tensor and writes

$$G^{ab} = -8\pi G\Theta^{ab}.$$  

Because of this equation $\Theta^{ab}$ is “conserved”, or more precisely, $\nabla_a \Theta^{ab} = 0$.


7. Also calculate the frequency spectrum for a particle moving as a harmonic oscillator with frequency $\omega_0$ as suggested in discussion 10 on page 204 of Barut. See Problem 14.4 in Jackson for more discussion.